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Elliptic functions and rings of integers, by Ph. Cassou-Noguès and M. J. Taylor. Progress in Mathematics, vol. 66, Birkhäuser, Boston, Basel and Stuttgart, 1987, xvi + 198 pp., $29.50. ISBN 0-8176-3350-2

The idea of constructing prescribed algebraic number fields by means of the values of real or complex functions is usually referred to as the 'Jugendtraum,' a word used by Kronecker in an 1880 letter to Dedekind [**8**]. Hilbert made the realization of this idea the twelfth problem of his 1900 address [7], adding that he considered "... this problem as one of the most profound and far-reaching in the theory of numbers and of functions." In the present book, Cassou-Noguès and Taylor give an exposition of Taylor's recent work on an "integral Jugendtraum" for the rings of integers of ray class fields of imaginary quadratic fields. The object of this work is to construct by means of elliptic functions explicit generators of such rings of integers either as algebras or as Galois modules.

The two examples which motivated the Jugendtraum were provided by the finite abelian extensions of either the rational numbers $\mathbf{Q}$ or of an imaginary quadratic field. By the Kronecker-Weber Theorem, every abelian extension of $\mathbf{Q}$ is contained in a cyclotomic field of the form $\mathbf{Q}(\zeta_n)$, where $\zeta_n = \exp(2\pi i/n)$ is a primitive nth root of unity for some positive integer n. Thus the values of the function $e(z) = \exp(2\pi i z)$ at rational z generate all abelian extensions of $\mathbf{Q}$. One can view these z as the points of finite order on the one-dimensional real torus $\mathbf{R}/\mathbf{Z}$. Suppose now that $\mathbf{Q}$ is replaced by an imaginary quadratic field K. In this case the torus $\mathbf{R}/\mathbf{Z}$ may be replaced by a two-dimensional torus $\mathbf{C}/\Omega$, where Ω is a nonzero ideal of $\mathscr{O}_K$ and we view K as a fixed subfield of $\mathbf{C}$. An elliptic function for $\mathbf{C}/\Omega$ is a meromorphic function of $z \in \mathbf{C}$ whose value at z depends only on $z \bmod \Omega$. A division point of $\mathbf{C}/\Omega$ is a point of finite order on $\mathbf{C}/\Omega$. By work of Weber, Feuter and Hasse, there are elliptic functions for $\mathbf{C}/\Omega$ such that every abelian extension of K is contained in a ray class field generated over K by the values of these functions at suitable division points. There are different combinations of elliptic functions which generate the