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Global analysis on foliated spaces by Calvin C. Moore and Claude Schochet. Mathematical Sciences Research Institute Publications, vol. 9, Springer-Verlag, New York, Berlin, Heidelberg, 1988, 377 pp., $34.00. ISBN 0-387-96664-1

Let M be a closed compact oriented Riemannian surface. The Euler number of M, $\chi(M)$, is given by $\sum(-1)^i \dim H^i(M;R)$, where $H^i(M;R)$ is the ith de Rham cohomology group of M. This integer determines M up to diffeomorphism. The Gauss-Bonnet theorem states that

$$\chi(M) = \int_M (1/2\pi)\Omega$$

where Ω is the curvature of the Levi-Civita connection on the tangent bundle of M. Now $d[(1/2\pi)\Omega] = 0$ so it defines a cohomology class on M, its Euler class, and we may interpret the theorem as saying that the Euler number of M (an analytic invariant) is given by the integral over M of a certain characteristic cohomology class (a topological invariant).

This simple theorem is at once the genesis and a paradigm for the index theory of elliptic operators, a theory which relates topological invariants of differential structures on the one hand to analytical invariants on the other. The central theorem in this theory is the Atiyah-Singer index theorem [AS]. Briefly, it says the following. Let M be a closed compact oriented manifold, let $E = (E_0, E_1, \dots, E_k)$ be a family of complex vector bundles over M, and let $d = (d_0, d_1, \dots, d_{k-1})$ be a family of differential operators, d_i mapping sections of E_i to sections of E_{i+1}. Suppose that $d_i \cdot d_{i-1} = 0$ and that the differential complex (E, d) satisfies a technical condition called ellipticity. Roughly speaking, ellipticity means that the associated Laplacians (see below) differentiate in all possible directions. Ellipticity implies, among other things, that for all i, $H^i(E, d) = \ker d_i / \operatorname{image} d_{i-1}$ is a finite dimensional vector space. Define the index of (E, d) to be

$$I(E, d) = \sum_{i=0}^{k} (-1)^i \dim H^i(E, d).$$