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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 20, Number 1, January 1989

0273-0979/89 $1.00 + $.25 per page

Dimensions of ring theory, by Constantin Nastasescu and Freddy van Oystaeyen. D. Reidel Publishing Company, Dordrecht, Boston, Lancaster and Tokyo, 1987, xi + 360 pp., $74.00. ISBN 90-277-2461-x

While mathematics is certainly not a "science of measurement", mathematicians do seek to "take the measure" of everything they study, often in the form of numerical, cardinal, or ordinal invariants—for example, to measure the deviation of a certain system from some ideal situation, to measure how likely or unlikely a certain object is to enjoy a certain property, or simply to measure the progress in some inductive procedure. Many such invariants have evolved, either directly or through analogy, from the Euclidean dimensions with which we measure our "real" world, and thus many invariants are called "dimensions" of some sort, usually decorated with one or more adjectives. Ring theory has its share of such dimensions, attached to both rings and modules, and since these dimensions have evolved in an algebraic rather than a geometric environment, their connection with Euclidean dimension may not be readily apparent. To illustrate, we discuss three examples—Goldie dimension, Krull dimension, and Gelfand-Kirillov dimension.

Goldie dimension. Vector space dimension cannot be applied directly to arbitrary modules because most modules do not have bases, and even among those that do (namely the free modules), one can find modules in which different bases may have different cardinalities. The dimension of a vector space V can, however, be expressed as the number of terms in a decomposition of V into a direct sum of irreducible subspaces, or as the maximum number of terms occurring in decompositions of V into direct sums of nonzero subspaces. Since the complexity of a module need not be reflected by direct sum decompositions, one looks at decompositions of submodules along with decompositions of a given module. Thus the *Goldie dimension* (also called the *uniform dimension*, the *uniform rank*, or the *Goldie rank*) of a module M is defined to be the supremum of the number of nonzero terms in any direct sum decomposition of any submodule of M.

This dimension arose in Goldie's 1958 development of noncommutative rings of fractions [3], since one necessary condition for a ring R to have a simple artinian ring of fractions Q is that the Goldie dimension of R (considered as a module over itself) be finite. More specifically, such a Q must be isomorphic (by the Artin-Wedderburn Theorem) to the ring