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OPERATOR THEORY AND ALGEBRAIC GEOMETRY

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Operator theory as the study of bounded linear operators on a complex Hilbert space is nearing the end of its first century. To date most effort has been directed toward the study of a single operator or of a selfadjoint algebra of operators. The work has relied not only on measure theory and functional analysis but on techniques from complex variables, topology, and algebra. But for single operator theory the topology is either planar or general, it is one complex variable, and it is linear algebra or the algebra of polynomials in one variable that is used. For operator algebras the relevant mathematics is more sophisticated and draws on increasing amounts of topology and geometry to the point that one has begun in the last decade to refer to parts of the study of operator algebras as "noncommutative topology and geometry."

In recent years the study of nonselfadjoint operator algebras has also enjoyed considerable success but this development has largely excluded spectral theory. The work of Carey and Pincus **[10]** is an exception. Multivariable spectral theory could be viewed by analogy as "noncommutative algebraic geometry," and such a development was the goal of the module approach to multivariable operator theory presented in **[13]**. The intent was to introduce methods from several variables algebra into operator theory. In this note we announce several results in multivariable operator theory whose proofs rely on techniques which are drawn from algebraic geometry or commutative algebra. Complete details will appear elsewhere.

An operator T on the complex Hilbert space **H** is said to be hyponormal if the self-commutator $[T^*, T] = T^*T - TT^*$ is positive definite. Any operator T makes **H** into a module over the algebra of polynomials $\mathbf{C}[z]$. A little reflection shows that **H** is a module over $\mathrm{Rat}(\sigma(T))$, the algebra of rational functions on the spectrum $\sigma(T)$ of T with poles off $\sigma(T)$. In **[6]** Berger and Shaw showed that if T is hyponormal and **H** is a finitely generated $\mathrm{Rat}(\sigma(T))$-module, then $[T^*, T]$ is trace class. Hence T is essentially normal and defines an element $[T]$ in $\mathrm{Ext}(\sigma(T)) = K_1(\sigma(T))$ **[7]**. The trace class commutator enables one to define the Chern character of this class following Helton and Howe **[16]**, Carey and Pincus **[10]**, and Connes **[11]**. Attempts at generalizing the Berger-Shaw result to several variables have failed although the Chern-Weil type construction of Carey-Pincus

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