Ramanujan has left a legacy that will keep mathematicians busy for many more decades.

ADDED IN PROOF (MAY 23, 1988). The above conjecture has been proven by Dean Hickerson.

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 19, Number 2, October 1988

0273-0979/88 $1.00 + $.25 per page

*Topological graph theory*, by Jonathan L. Gross and Thomas W. Tucker, John Wiley and Sons, New York, Chichester, Brisbane, Toronto, Singapore, xv + 351 pp., $59.95. ISBN 0-471-04926-3

Topological graph theory began with the 1890 paper of Heawood [**1**] in which it was pointed out that Kempe's proof of the 4-colour theorem was incorrect. Heawood proved instead that every map on $S_h$, the sphere with $h$ handles attached, can be coloured in $\chi(S_h) = [\frac{1}{2}(7+\sqrt{1+48h})]$ colours for each $h \geq 1$. He claimed that this is best possible since a map with $\chi(S_h)$ pairwise adjacent countries (or, equivalently, a complete graph with $\chi(S_h)$ vertices) can be drawn on $S_h$ for each $h \geq 1$. While this claim, which became known as the Heawood conjecture, is correct, it took almost 80 years until a proof was completed. The main ideas and the major part of the proof were provided by G. Ringel who wrote a book on the proof [**2**]. The final cases of the proof were done by Ringel and Youngs.

The Heawood conjecture led to the following general question: Given a graph $G$ and a natural number $h$, can $G$ be embedded into $S_h$? While this problem is $NP$-complete, (and thus probably hopeless), as shown recently by the reviewer, there are many results for special classes of graphs. Most of the investigations motivated by the Heawood conjecture are concerned with the existence and properties of certain embeddings. However, the recent Robertson-Seymour theory on minors has shown that topological graph theory is also important as a tool and has a natural place in general discrete mathematics. One of the highlights in the Robertson-Seymour theory is the following: Let $p$ be a graph property which is preserved under minors, that is, if $G$ has property $p$ and $H$ is obtained from $G$ by deleting or contracting edges, then also $H$ has property $p$. Then there exist only finitely many minor-minimal graphs that do not have property $p$, and there exists a polynomially bounded algorithm for deciding if a graph has property $p$. In order to understand the proof of this general result it is necessary to be familiar with some topological graph theory.