

at home on personal computers. It really would not be that difficult, starting from the database the editors of this work have painstakingly assembled. In other words, what we have here are six lovely oranges, but what I want is just one Apple.

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Differential geometry of complex vector bundles, by Shoshichi Kobayashi. Publications of the Mathematical Society of Japan, no. 15 Iwanami Shoten Publishers and Princeton University Press, Princeton, N. J., 1987, xi+304 pp., \$57.50. ISBN 0-691-08467-x

The book under review is a research monograph laying the foundation for the theory of Einstein-Hermitian structures on holomorphic vector bundles. The concept of an Einstein-Hermitian structure has been introduced by the author in 1978 and has proved to be very fundamental and popular since. Being fundamental usually is not sufficient for being popular; what made this concept so popular? I see at least two principal reasons:

The first is that it provides a link between differential geometry and algebraic geometry, leading to a good problem, the so-called Kobayashi-Hitchin conjecture. This problem has in the meantime been completely solved by Donaldson [2, 3] and Uhlenbeck and Yau [10].

The second reason is that the solution of this conjecture in the 2-dimensional case made several spectacular applications possible.

In complex dimension 2 the conjecture ties Yang-Mills theory and algebraic geometry. Together with Donaldson's fundamental work on instanton moduli spaces it led to unexpected results on the differential topology of algebraic surfaces [4, 5, 9].

What is an Einstein-Hermitian structure? To explain this consider a compact complex submanifold $X \subset \mathbf{P}_{\mathbf{C}}^N$ of some projective space endowed with the induced metric. A holomorphic vector bundle \mathcal{E} on X is a locally trivial fibre space over X with fibres \mathbf{C}^r and holomorphic transition functions. Suppose we want to equip \mathcal{E} with a Hermitian structure h , i.e., a C^∞ family $(h(x))_{x \in X}$ of Hermitian metrics on the fibres. It would then be natural to look for a best or in some sense distinguished structure h . Now it is a fundamental fact that every choice of a Hermitian structure on a holomorphic bundle gives rise to an associated concept of parallelism, in other words, to a compatible connection D_h in \mathcal{E} . The mean curvature K_h of this connection is a Hermitian form on \mathcal{E} , also depending on the metric on X , which measures how the bundle is twisted. If this form is proportional to the metric h , $K_h = c \cdot h$, one says that h is an Einstein-Hermitian structure or that (\mathcal{E}, h) is an Einstein-Hermitian bundle.

This concept obviously generalizes the notion of a Kähler-Einstein metric. Kobayashi arrived at the definition of an Einstein-Hermitian structure when