

compactified moduli space, spaces of Kleinian groups, and applications to topology, geometry, and physics, in addition to some of the topics discussed in the last paragraph) is a sign of the vitality of the subject.

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Differential systems and isometric embeddings, by Phillip A. Griffiths and Gary R. Jensen. *Annals of Mathematics Studies*, vol. 114, Princeton University Press, Princeton, N. J., 1987, xii+225 pp., \$35.00 (cloth). \$15.00 (paper). ISBN 0-691-08429-7

A submanifold of any Euclidean space has an induced metric structure, and it is natural to wonder about the converse: Can a given Riemannian structure on a manifold M^n be induced by an embedding of M into some Euclidean space E^N ? The search for local and global isometric embeddings has been fruitful for mathematics. However the three major advances in the local theory have been relatively inaccessible even to many workers in the field.

In terms of partial differential equations, the local embedding problem reduces to solving

$$(1) \quad \sum_{\nu=1}^N \frac{\partial u^\nu}{\partial x_i} \frac{\partial u^\nu}{\partial x_j} = g_{ij},$$

where (g_{ij}) is a positive definite, symmetric $n \times n$ matrix.

When $n = 1$ such a solution clearly exists, and one may take $N = 1$. For $n = 2$ there are many special results, some of which are old [We] and others quite recent [Li]. The basic question is still open: Can every two-dimensional C^∞ Riemannian manifold be locally isometrically embedded in E^3 ?

Notice that (1) becomes determined, in the sense that the number of equations equals the number of unknowns, when $N = \frac{1}{2}n(n+1)$. Most of our discussion will be for this dimension. Also notice that not even the case where g_{ij} is real analytic is immediate. The difficulty, in classic PDE terms,