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TOPOLOGICAL TYPES AND MULTIPLICITIES OF ISOLATED QUASI-HOMOGENEOUS SURFACE SINGULARITIES

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ABSTRACT. Two germs of 2-dimensional isolated quasi-homogeneous hypersurface singularities have the same topological type if and only if they have the same characteristic polynomial and the same fundamental group for their links. In particular, multiplicity is an invariant of topological type, an affirmative answer to Zariski's question in this case.

Let $(V,0)$ and $(W,0)$ be germs of isolated hypersurface singularities in $\mathbf{C}^{n+1}$. We say that $(V,0)$ and $(W,0)$ have the same topological type if there is a germ of a homeomorphism from $(\mathbf{C}^{n+1},V,0)$ to $(\mathbf{C}^{n+1},W,0)$. In his retiring Presidential address to the American Mathematical Society in 1971, Zariski asked whether $(V,0)$ and $(W,0)$ have the same multiplicity if they have the same topological type. He expected that topologists would be able to answer his question in relatively short order. However the question appears to be much harder than what Zariski thought. Even special cases of Zariski's problem have proved to be extremely difficult. Only recently Greuel [**4**] and O'Shea [**14**] proved independently that topological type constant families of isolated quasi-homogeneous singularities are equimultiple. For quasi-homogeneous surface singularities, Laufer [**5**] explained the constant multiplicity for a topological type constant family of singularities from a different viewpoint. However it is not known whether two quasi-homogeneous singularities having the same topological type can be put into a topological type constant family. Let $(V,0)$ be a dimension two isolated hypersurface singularity. Lê and Teissier [**8**] observed that A'Campo's work [**1**] can often be used to give positive results towards Zariski's question. Let $C(V,0)$ be the reduced tangent cone. Let $\mathbf{P}C(V,0)$ denote the hypersurface in $\mathbf{CP}^2$ over which $C(V,0)$ is a cone. Then, the work of A'Campo shows that the multiplicity of $(V,0)$ is determined by the topological type of $(V,0)$ in case the topological Euler number $\chi(\mathbf{P}C(V,0))$ is nonzero. The same arguments also show that, for isolated hypersurface two-dimensional singularities, the embedded topology and the multiplicity determine $\chi(\mathbf{P}C(V,0)))$. However, so far, by using A'Campo's result, one can only prove that a surface in $\mathbf{C}^3$ having at 0 a singularity of multiplicity 2 cannot have the same topological type at 0 as another surface of multiplicity different from 2.

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