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FROM VANISHING THEOREMS TO ESTIMATING THEOREMS: THE BOCHNER TECHNIQUE REVISITED

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Dedicated to Marcel Berger on his 60th birthday

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Introduction. In the 1940s, S. Bochner devised an analytic technique to obtain *vanishing theorems* for some topological or geometric invariants (e.g. Betti numbers, the dimension of the vector space of Killing vector-fields) on a closed (i.e. compact without boundary) Riemannian manifold, under some curvature assumption.

As a matter of fact, the word technique might be misleading. On the one hand it is not so easy to explain the technical details of the proofs in which S. Bochner's ideas are used and this is not our purpose here; on the other hand, the ideas are quite simple. Indeed, the basic idea is to show that some object (a harmonic form in the case of Betti numbers, a Killing vector-field,...) satisfies an elliptic inequality, provided that some curvature assumption is satisfied. The proofs then reduce to applying the maximum principle or to integrating over the manifold.

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