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BAND-LIMITED FUNCTIONS: L^p-CONVERGENCE

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I. Introduction and statement of results. In this paper we shall be concerned with a set of functions, namely the prolate spheroidal wave functions, which have been considered by several authors especially in relation with problems of communication theory. These functions appear naturally in the formulation of the uncertainty principle: a very interesting problem there is that of making signals $f = f(t)$ of total power $\|f\|_2 = 1$ with both

$$\tau^2 = \int_{-T}^{+T} |f(t)|^2\, dt \qquad \omega^2 = \int_{-\Omega}^{+\Omega} |\hat{f}(\xi)|^2\, d\xi$$

as close to 1 as possible for given positive numbers T and Ω, where

$$\hat{f}(\xi) = \int_{-\infty}^{+\infty} e^{-2\pi i x\cdot\xi} f(x)\, dx$$

denotes the Fourier transform.

$\tau = 1$ means that the signal is confined to the period $|t| \leq T$ or that f is time-limited.

$\omega = 1$ means that its power spectrum is confined to the band $|\xi| \leq \Omega$ or that f is band-limited.

It is a well-known fact about the Fourier transform that both τ and ω cannot be equal to 1 at the same time. In a series of papers by Landau, Pollak, and Slepian [**4, 5, 7, 8**] the following result was proved: the pairs (τ, ω) corresponding to signal f with $\|f\|_2 = 1$ describe the region defined by the inequalities

$$\cos^{-1}(\tau) + \cos^{-1}(\omega) \geq \cos^{-1}\sqrt{\lambda_0}, \qquad 0 \leq \tau \leq 1, 0 \leq \omega \leq 1$$

(with the proviso that if τ or $\omega = 1$ $(= 0)$, then the other is > 0 (< 1)) where

$$\lambda_0 = \sup\left\{ \int_{-T}^{+T} |f(t)|^2\, dt | f \in L^2(\mathbf{R}), \|f\|_2 = 1, \operatorname{supp}(\hat{f}) \subset [-\Omega, +\Omega] \right\}.$$

Another interesting result is the so-called dimension theorem. Roughly speaking it says that the dimension of the set of signals band-limited to the interval $[-\Omega, +\Omega]$ and "concentrated" in $[-T, +T]$ is $4T\Omega$. We refer to the above-mentioned papers for a more precise statement.

In the proof of these results, the operator

$$Tf(x) = \int_{-T}^{+T} \frac{\sin 2\pi\Omega(t-s)}{\pi(t-s)} f(s)\, ds$$

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