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Projective representations of finite groups, by Gregory Karpilovsky, Marcel Dekker, Inc., New York and Basel, 1985, xiii + 644 pp., $89.75. ISBN 0-8247-7313-6

Projective representations take their name from projective geometry. To be specific, let G be a finite group, K a field, and V a finite-dimensional vector space over K. Let h be a homomorphism of G into the projective general linear group $\mathrm{PGL}(V)$, i.e., the group of all projective transformations of the projective space whose points are the one-dimensional subspaces of V. Since many of the finite simple groups are defined as subgroups of groups $\mathrm{PGL}(V)$ for finite K, their natural injections into $\mathrm{PGL}(V)$ furnish important examples. $\mathrm{PGL}(V)$ can be identified with the quotient group of the group $\mathrm{GL}(V)$ of all invertible linear transformations of V by the normal subgroup Z consisting of scalar multiples of the identity $1_{\mathrm{GL}(V)}$ by the elements of $K^\times = K - \{0\}$. Accordingly, h can be studied as follows: for each $g \in G$ choose a representative $\rho(g)$ of the coset $h(g)Z = h(g)K^\times$; then ρ is a mapping of G into $\mathrm{GL}(V)$ such that

$$\rho(g_1)\rho(g_2) = \alpha(g_1, g_2)\rho(g_1 g_2) \tag{1}$$

for some mapping α of $G \times G$ to $K^\times$; we can suppose that

$$\rho(1_G) = 1_{\mathrm{GL}(V)}. \tag{2}$$

Then ρ can be studied in place of h; this replaces a projective situation by a more familiar linear one, though at the price that ρ depends on arbitrary choices. Any mapping ρ of G to $\mathrm{GL}(V)$ that satisfies (1) and (2) for any α is called a *projective representation* of G; if α is specified, ρ is called an *α-representation.*

Many examples can be constructed as follows: let

$$1 \to A \to H \xrightarrow{f} G \to 1 \tag{3}$$

be a *central extension*, i.e., an epimorphism $H \to G$ of finite groups with $\ker f \cong A$ contained in the center of H; thus $G \cong H/A$ if we identify $\ker f$ and A. (The group H is also called a central extension of G.) For each $g \in G$ choose an inverse image $\mu(g) \in H$ such that $f(\mu(g)) = g$, with $\mu(1_G) = 1_H$. Then for each linear representation r of H, the rule

$$\rho(g) = r(\mu(g)) \tag{4}$$

defines a projective representation ρ of G. For example, if H is either of the nonabelian groups of order 8, A its center, and f the natural map to $G = H/A$, the 2-dimensional irreducible complex representation of H yields a projective representation of the Klein four-group. Central extensions play an important role in the proof of the classification of finite simple groups [**2**; **7**, pp. 295–303]; furthermore, attempts to use the classification to prove a conjecture for arbitrary finite groups sometimes reduce the conjecture to the