11. ______, *The quadratic sieve factoring algorithm*, Advances in Cryptology (T. Beth, N. Cot and I. Ingemarsson, eds.), Springer Lecture Notes in Computer Science **209** (1985), 169–182.

12. ______, *Fast, rigorous factorization and discrete logarithm algorithms*, Discrete Algorithms and Complexity (D. S. Johnson, T. Nishizeki, A. Nozaki and H. S. Wilf, eds.), Academic Press, Orlando, Florida, 1987, pp. 119–143.

13. M. O. Rabin, *Probabilistic algorithms for testing primality*, J. Number Theory **12** (1980), 128–138.

14. J. B. Rosser and L. Schoenfeld, *Approximate formulas for some functions of prime numbers*, Illinois J. Math. **6** (1962), 64–94.

15. R. J. Schoof, *Elliptic curves over finite fields and the computation of square roots* mod p, Math. Comp. **44** (1985), 483–494.

16. R. Solovay and V. Strassen, *A fast Monte-Carlo test for primality*, SIAM J. Comput. **6** (1977), 84–85; erratum, ibid. **7** (1978), 118.

17. V. Strassen, *Einige Resultate über Berechnungskomplexität*, Jahresber. Deutsch. Math.-Verein **78** (1976/77), 1–8.

CARL POMERANCE

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 18, Number 1, January 1988

0273-0979/88 $1.00 + $.25 per page

Monotone iterative techniques for nonlinear differential equations, by G. S. Ladde, V. Lakshmikantham, and A. S. Vatsala, Pitman Publishing Company, Boston, London, Melbourne, 1985, x + 236 pp., $110.00. ISBN 0-273-08707-X

The monotone method and its associated upper-lower solutions for nonlinear partial differential equations have been given extensive attention in recent years. The method is popular because not only does it give constructive proof for existence theorems but it also leads to various comparison results which are effective tools for the study of qualitative properties of solutions. The monotone behavior of the sequence of iterations is also useful in the treatment of numerical solutions of various boundary value and initial boundary value problems. Recognizing its immense value to nonlinear problems, the authors repeatedly apply the monotone method and the idea of upper-lower solutions to various first- and second-order partial differential equations. To illustrate the basic idea of the monotone method, let us consider a typical elliptic boundary value problem in the form

$$-L[u] = f(x,u) \text{ in } \Omega, \qquad B[u] = h(x) \text{ on } \partial\Omega,$$

where L is a uniformly elliptic operator in a bounded domain Ω and B is a linear boundary operator on $\partial\Omega$. Suppose there exists an ordered pair of upper and lower solutions v and w, that is, v and w are smooth functions with $v \geqslant w$ such that

$$-L[v] \geqslant f(x,v) \text{ in } \Omega, \qquad B[v] \geqslant h(x) \text{ on } \partial\Omega,$$