BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 18, Number 1, January 1988

VIRTUAL COHOMOLOGICAL DIMENSION OF MAPPING CLASS GROUPS OF 3-MANIFOLDS

DARRYL McCULLOUGH

The mapping class group of a topological space is the group of self-homeomorphisms modulo the equivalence relation of isotopy. For 2-manifolds (of finite type), it is a discrete group which is known (see [**M**, **H1**, **H2**, **H3**, **H4**]) to share many of the properties of arithmetic subgroups of linear algebraic groups, although it is not arithmetic. In this note we describe the results of [**M1**], which show that the mapping class groups of many 3-manifolds share some of these properties.

More precisely, a group G is said to be *of type FL* if the trivial G-module $\mathbf{Z}$ admits a resolution of finite length by finitely generated free G-modules (see [**S**]), and is said to be a *duality group* when there is a G-module C such that cap product with an element $e \in H_n(G; C)$ induces isomorphisms $H^k(G; A) \cong H_{n-k}(G; C \otimes A)$ for all k and A (see [**B-E**]). The classes of groups of type FL and duality groups are closed under extension. The main result of [**M1**] is

THEOREM 1. *Let M be a compact orientable irreducible sufficiently large 3-manifold. Then the mapping class group $\mathcal{H}(M)$ is finitely presented and contains a subgroup of finite index which is of type FL. If the boundary of M is incompressible, then the subgroup is a duality group.*

The finite presentation of $\mathcal{H}(M)$ in the boundary-incompressible case follows from work of Johannson [**J**] and Hemion [**H5**] (see [**W**]). In the case of compressible boundary, it was proved by R. Kramer for handlebodies and by P. Grasse [**G**] in general.

A finitely presented group of type FL is also called a *geometrically finite* group because it is the fundamental group of a finite aspherical complex. Its cohomology, with any coefficient module, vanishes above a certain dimension, called the *cohomological dimension* of the group.

When a group G contains a subgroup of finite index which has finite cohomological dimension, G is said to have finite virtual cohomological dimension. This dimension is well defined (see [**S**]) and is denoted by $\dim(G)$.

1. The proof of Theorem 1. The characteristic submanifold theory due to Johannson [**J**] and Jaco and Shalen [**J-S**] shows that Haken 3-manifolds

Received by the editors July 27, 1987.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 57M99; Secondary 57S99, 55S37, 20F34.

Key words and phrases. 3-manifold, mapping class group, homeomorphism, diffeomorphism, cohomological dimension, duality group, FL, VFL.

Supported in part by NSF Grants DMS-8420067 and DMS-8701666.