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CLASSICAL INVARIANT THEORY AND THE EQUIVALENCE PROBLEM FOR PARTICLE LAGRANGIANS

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ABSTRACT. The problem of equivalence of binary forms under the general linear group is shown to be a special case of the problem of equivalence of particle Lagrangians under the pseudogroup of transformations of both the independent and dependent variables. The latter problem has a complete solution based on the equivalence method of Cartan. This leads to the determination of a *universal function* which relates two particular rational covariants of any binary form. In essence, the main result is that two binary forms are equivalent if and only if their universal functions are identical. Extensions to forms in higher dimensions are indicated.

Consider a first-order variational problem

$$\mathcal{L}[u] = \int L(x,u,p)\,dx, \qquad x,u \in \mathbf{R},\ p \equiv \frac{du}{dx},$$

where the *Lagrangian* $L(x,u,p)$ is analytic on a domain $\Omega \subset \mathbf{R}^3$. Two Lagrangians L and $\tilde{L}$ are *equivalent* if there exists a change of variables $\tilde{x} = \varphi(x,u)$, $\tilde{u} = \psi(x,u)$ mapping one to the other. The change in the derivative is a linear fractional transformation

$$\tilde{p} = \frac{a \cdot p + b}{c \cdot p + d}, \tag{1}$$

where $a = \psi_u$, $b = \psi_x$, $c = \varphi_u$, $d = \varphi_x$. Equivalent Lagrangians must be related by

$$L(x,u,p) = (c \cdot p + d) \cdot \tilde{L}(\tilde{x},\tilde{u},\tilde{p}). \tag{2}$$

(This equivalence problem is a restricted version of the "true" Lagrangian equivalence problem, in which one can also add in a divergence term, solved in [**7**].)

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