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Theory of limit cycles, by Ye Yan-qian, Cai Sui-lin, Chen Lan-sun, Huang Ke-cheng, Luo Ding-jun, Ma Zhi-en, Wang Er-nian, Wang Ming-shu, and Yang Xin-an; translated by Chi Y. Lo. Translations of Mathematical Monographs, Vol. 66, American Mathematical Society, Providence, R. I., 1986, xi + 435 pp., $140.00. ISBN 0-8218-4518-7

The problem of the existence of limit cycles for certain nonlinear differential equations is one of the most fundamental areas of research in differential equations. This question has intrigued mathematicians since the discovery of Henri Poincaré [4] that certain nonlinear systems do in point of fact admit an oscillatory behavior. Indeed this work has motivated the creation of the fields of topological dynamics and general stability theory.

Part of the Hilbert sixteenth problem set forth at the 1900 International Congress of Mathematicians was concerned precisely with this topic. Explicitly, David Hilbert posed the problem of finding the maximum number of limit cycles for a first-order differential equation of the form

$$dy/dx = P(x, y)/Q(x, y), \tag{$*$}$$

where $P(x, y)$ and $Q(x, y)$ are polynomials. To give the uninitiated reader an idea of the difficulty of this still unsolved problem, we should note that two decades passed after the problem was initially posed before the mathematician Henri Dulac [2] succeeded in proving that such an equation admitted only a finite number of limit cycles. (The book under review has an extensive list of references where the interested reader can find more details about the above works.) Even in the case in which $P(x, y)$ and $Q(x, y)$ are quadratic polynomials this problem still remains unsolved. The maximal number of limit cycles for such quadratic systems has been conjectured to be 3, 4, and 5 at various times. (See [5] and the references therein.)

From a more applied point of view, oscillatory phenonema have been discovered in great abundance in "nature", perhaps some of the most striking occurring in electrical engineering, and in particular in electronics. It was B. van der Pol [6] who first wrote down, in the mid-1920s, a nonlinear differential equation to describe the stable oscillations in the triode vacuum tube which had a tremendous impact on the field of electronics design and of course on applied mathematics. (It was actually the physicist A. Andronov [1] who proved that the closed isolated trajectory discovered by van der Pol was indeed a limit cycle in the sense of Poincaré.) It was van der Pol's discovery that provided a major impetus to the study of nonlinear ordinary differential equations in both engineering and mathematics, and in particular, to the nonlinear phenomenon of limit cycles. We should add that the van der Pol equation has become a standard topic in modern engineering electronics courses as well as in differential equations.

The book under review covers some of the basic material on the theory and properties of limit cycles, e.g., the Poincaré-Bendixson theorem, multiplicity