BOOK REVIEWS

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Finite groups of Lie type. Conjugacy classes and complex characters, by Roger W. Carter. John Wiley and Sons, Chichester, New York, Brisbane, Toronto and Singapore, 1985, xii + 554 pp., $69.95. ISBN 0-471-90554-2

The finite groups of Lie type are the finite analogues of the reductive Lie groups. The importance of these finite groups for the general theory of finite groups is shown by the classification of finite simple groups, according to which any finite noncyclic simple group which is not an alternating group or one of 26 sporadic groups is a composition factor of some finite group of Lie type.

Among the finite groups of Lie type occur classical groups over a finite field $\mathbf{F}_q$, such as the general linear group $\mathrm{GL}_n(\mathbf{F}_q)$ of all nonsingular $n \times n$ matrices over $\mathbf{F}_q$, the special orthogonal groups, and the symplectic groups over $\mathbf{F}_q$. One encounters such groups in the very beginning of the theory of finite groups: Galois in 1832 knew already the general linear groups over the prime fields. Subsequently, C. Jordan (around 1870) studied classical groups over finite fields. His work was continued by L. E. Dickson (in the beginning of this century). In their work they discussed primarily group-theoretical questions like the description of normal subgroups and of simple composition factors. Their methods were those of linear algebra.

The more recent developments of the theory of finite groups of Lie type can all be viewed as being due to invasions of the theory by other branches of mathematics.

These started in 1955, when C. Chevalley introduced ideas from the theory of complex semisimple Lie algebras. He constructed, for each simple Lie algebra (by a reduction modulo p procedure), a matrix group over $\mathbf{F}_q$, which led to a corresponding finite simple group. These matrix groups have many features in common with semisimple Lie groups. Somewhat later, in the sixties, the insight came that the best way to deal with Chevalley's groups is to view them in the context of the theory of linear algebraic groups. This theory was initiated by A. Borel in 1956 and further developed soon after. One is then led naturally to the use of ideas and methods from algebraic geometry, over fields of characteristic $p > 0$. In that context the definition of a finite group of Lie type is as follows.

Let $\overline{\mathbf{F}}_q$ be an algebraic closure of $\mathbf{F}_q$. An algebraic group over $\mathbf{F}_q$ (briefly: an $\mathbf{F}_q$-group) is a subgroup of some $\mathrm{GL}_n(\overline{\mathbf{F}}_q)$ whose elements are precisely the