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BMO ON THE BERGMAN SPACES OF THE CLASSICAL DOMAINS

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Let Ω be a bounded symmetric (Cartan) domain with its Harish-Chandra realization in $\mathbf{C}^n$ [**T**]. For dv the usual Euclidean volume measure on $\mathbf{C}^n = \mathbf{R}^{2n}$, normalized so that $v(\Omega) = 1$, we consider the Hilbert space of square-integrable complex-valued functions $L^2 = L^2(\Omega, dv)$ and the Bergman subspace $H^2 = H^2(\Omega)$ of holomorphic functions in L^2. The self-adjoint projection from L^2 onto H^2 is denoted by P. For f, g in L^2, we consider the *multiplication operator* M_f on L^2 given by $M_f g = fg$ and the *Hankel operator* H_f on L^2 given by $H_f = (I-P)M_fP$. For f in L^2, these operators are only densely defined and may be unbounded. The commutator $[M_f, P] = M_fP - PM_f$ is densely defined on L^2 and may also be unbounded. From the equations

$$[M_f, P] = H_f - H_{\bar f}^*, \quad (I-P)[M_f,P] = H_f, \quad [M_f,P](I-P) = -H_{\bar f}^*,$$

it follows that $[M_f, P]$ is a bounded operator if and only if $H_f, H_{\bar f}$ are bounded. Moreover, $[M_f, P]$ is a compact operator if and only if $H_f, H_{\bar f}$ are compact.

In earlier work [**BCZ**], it was shown that for f in $L^\infty(\Omega)$, the algebra of bounded measurable functions on Ω, $[M_f, P]$ is compact if and only if f has vanishing mean oscillation at the boundary $\partial\Omega$, where oscillation is defined in terms of the Bergman metric on Ω. In this note, we announce the companion result: *For f in L^2, $[M_f, P]$ is bounded if and only if f is of "bounded mean oscillation on Ω"*, where oscillation is defined as in [**BCZ**]. The space of such functions is denoted by BMO(Ω). We also obtain the expected result that: *For f in L^2, $[M_f, P]$ is compact if and only if f is in the subspace* $\mathrm{VMO}_\partial(\Omega)$ *of functions which have vanishing mean oscillation at the boundary* $\partial\Omega$. Our results are analogous to known results for arc-length measure on the unit circle [**G**, p. 278] and demonstrate the value of the Bergman metric in function-theoretic analysis on the classical domains.

Let $K(\cdot, a)$ be the Bergman reproducing kernel in $H^2(\Omega)$ for evaluation at $a \in \Omega$. For

$$k_a(\cdot) = K(a,a)^{-1/2}K(\cdot, a),$$

we define the *Berezin transform* of f in L^2 [**BCZ**] by

$$\tilde{f}(a) = \langle f k_a, k_a \rangle$$

where $\langle \cdot, \cdot \rangle$ is the usual L^2 inner product. For typographical reasons, we write the Berezin transform of $|f|^2$ as $(|f|^2)^\sim$. It follows from known properties of

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