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REPRESENTATIONS OF MOD p LIE ALGEBRAS

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Let G be a semisimple, simply connected algebraic group defined over an algebraically closed field k of characteristic $p > 0$. Because any rational G-module inherits the structure of a restricted module (in the sense of [**4**, p. 188]) for the Lie algebra $\mathcal{G}$ of G, the representation theory of $\mathcal{G}$ has primarily focused on the study of restricted modules. We outline here our recent investigations of the more general—that is, not necessarily restricted—represention theory of $\mathcal{G}$. Details will appear in [**3**].

We approach the representation theory of $\mathcal{G}$ through that of a family of finite-dimensional quotient algebras of the universal enveloping algebra $U(\mathcal{G})$ of $\mathcal{G}$. As described below, these algebras are parametrized by characters on a certain abelian subalgebra $\mathcal{O}$ of $U(\mathcal{G})$. Because the restricted enveloping algebra $V(\mathcal{G})$ appears as a distinguished member of this family (the others being thought of as "deformations" of $V(\mathcal{G})$), a better understanding of the representation theory of these algebras may lead to a clearer picture of that of $V(\mathcal{G})$.

For a restricted Lie algebra $\mathcal{G}$, we employ the central subalgebra $\mathcal{O} \subset U(\mathcal{G})$ considered by Zassenhaus in his foundational paper [**10**]. Namely, $\mathcal{O}$ is the image of the semilinear monomorphism $S^{\cdot}(\mathcal{G}) \to U(\mathcal{G})$ defined on the symmetric algebra $S^{\cdot}(\mathcal{G})$ of $\mathcal{G}$ by sending $X \in \mathcal{G}$ to $X^p - X^{[p]}$. Using the Jordan decomposition of the dual $\mathcal{G}^*$ given in [**7**], we obtain properties such as "regular", "semisimple", or "nilpotent" for characters $\chi\colon \mathcal{O} \to k$ whenever $\mathcal{G} = \mathrm{Lie}(G)$.

PROPOSITION 1. *Let $\mathcal{G}$ be a restricted Lie algebra of dimension d and let $\chi\colon \mathcal{O} \to k$ be a character with associated one-dimensional $\mathcal{O}$-module k_χ. Then $A_\chi \equiv U(\mathcal{G}) \otimes_{\mathcal{O}} k_\chi$ is a Frobenius algebra of dimension p^d. Moreover, for each irreducible $\mathcal{G}$-module M, there is a unique character $\chi\colon \mathcal{O} \to k$ such that the action of $U(\mathcal{G})$ on M factors through A_χ.*

Of course, if $\chi = 0$ then $A_\chi \cong V(\mathcal{G})$, the restricted enveloping algebra of $\mathcal{G}$.

One easily proves that $\mathrm{Ext}^{\cdot}_{U(\mathcal{G})}(M, M') = 0$ whenever M is an A_χ-module, M' is an $A_{\chi'}$-module, and $\chi \neq \chi'$. On the other hand, nontrivial computations are facilitated by the following spectral sequence.

PROPOSITION 2. *Let $\mathcal{G}$ and A_χ be as in Proposition 1. If M and N are two A_χ-modules with M finite-dimensional, then there is a natural spectral*

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