BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 17, Number 1, July 1987

THE DEGREE OF A SEVERI VARIETY

ZIV RAN

Consider the curves of some fixed degree d in the complex projective plane $\mathbf{P}^2$. These are parametrized by a projective space P_d, projectivization of the vector space of homogeneous polynomials of degree d in three variables. By a *Severi variety* we mean either the (locally closed) algebraic subset of P_d corresponding to curves with a fixed number of nodes (i.e., points where the curve has two transverse smooth branches) and no other singularities, or the subset of the latter corresponding to *irreducible* curves. These varieties have received considerable attention since they were introduced by Enriques [**1**] and Severi [**7**]. For an expository account, see [**2**] or [**5**]. For instance the famous *Severi problem*, recently solved by Harris [**3**], is to prove the former authors' claim that a Severi variety parametrizing irreducible curves is itself irreducible.

With the Severi problem now out of the way, the time seems ripe to begin asking some finer questions about the Severi varieties. One such question is: what is the degree, in P_d, of a Severi variety? The purpose of this announcement is to give a recursive procedure for answering this question. This procedure involves, even in its statement, a generalization of the Severi varieties to varieties of nodal curves on the blowup $\tilde{\mathbf{P}}^2$ of $\mathbf{P}^2$ at a point. We give a formula (see Theorem 2) for the degree of a generalized Severi variety for curves of given "type" (d, e), $0 \le e \le d-2$, and given number of nodes in terms of degrees of similar varieties for curves of various types (d', e') where $e' > e$ or $d' < d$. Since the degree of a generalized Severi variety for curves of type $(d, d-1)$ is easily computed, this gives an effective procedure for computing the degree of any generalized Severi variety. Among the generalized Severi varieties, the "ordinary" ones are those for type $(d, 0)$. Thus even if one only cares about the latter, he is led by the above procedure to the former.

1. Setup. Let $b\colon \tilde{\mathbf{P}}^2 \to \mathbf{P}^2$ denote the blowing up of a point, with exceptional divisor E, and for integers $d \ge e \ge 0$ let $L_{d,e}$ denote the line bundle $b^*\mathcal{O}(d)(-eE)$ on $\tilde{\mathbf{P}}^2$. For

$$-d+1 \le g \le \frac{(d-1)(d-2)}{2} - \frac{e(e-1)}{2}$$

(respectively,

$$0 \le g \le \frac{(d-1)(d-2)}{2} - \frac{e(e-1)}{2}),$$

denote by $V^{d,g,e}$ (resp. $V(d,g,e)$) the (locally closed) subset of the complete linear system $|L_{d,e}|$ consisting of (possibly reducible) (resp. irreducible) nodal

Received by the editors October 9, 1986 and, in revised form, February 10, 1987.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 14H10; Secondary 14D20.
Research partially supported by the NSF.