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GLOBALIZATIONS OF HARISH-CHANDRA MODULES

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1. Introduction. Let G be a reductive Lie group, $\mathfrak{g}$ the complexification of its real Lie algebra $\mathfrak{g}_0$, and K a maximal compactly embedded subgroup of G. In order to study various classes of representations (π, V_π) of G one makes use of the associated Harish-Chandra module (π_K, V). On the other hand, a Harish-Chandra module V can globalize to a representation of G in a variety of ways. Among these is the maximal globalization [**7**], which depends on V in a canonical manner. Here we show that several geometric constructions of representations of G are equivalent, all leading to the maximal globalization of certain standard derived functor modules.

For simplicity of exposition we describe our result only for groups G of Harish-Chandra class. It also holds for the larger class [**9**] of reductive groups.

2. The cohomologies. Consider the datum $(H, \chi, \mathfrak{b})$ where H is a θ-stable Cartan subgroup of $G, \mathfrak{b}$ is a Borel subalgebra of $\mathfrak{g}$ that contains $\mathfrak{h}$, and χ is a finite-dimensional irreducible representation of H. View the representation space E_χ as a $(\mathfrak{b}, H)$-module and let $\mathbf{E}_\chi$ denote the associated homogeneous vector bundle over G/H. Let $\mathfrak{n} = [\mathfrak{b}, \mathfrak{b}]$, the nilradical of $\mathfrak{b}$. One can attach a complex of $\mathbf{E}_\chi$-valued differential forms to the datum $(H, \chi, \mathfrak{b})$, for which the global sections are given by

$$(2.1) \qquad (C^\infty(G) \otimes E_\chi \otimes \Lambda\mathfrak{n}^*)^H, d.$$

Here H and $\mathfrak{n}$ act on $C^\infty(G)$ by right translation, the superscript H refers to the space of H-invariants, and d is the coboundary operator for Lie algebra cohomology of $\mathfrak{n}$.

REMARK 1. The sheaf version of the complex (2.1) is not, in general, acyclic. Thus (2.1) computes the hypercohomology of a complex of sheaves on G/H rather than the cohomology of a single sheaf.

By taking K-finite $\mathbf{E}_\chi$-valued differential forms whose coefficients are formal power series, one obtains a formal analog of (2.1). The cohomology groups of this formal complex are the standard Zuckerman modules $I^p(G/H, \mathbf{E}_\chi)$. See [**3**].

Let $\mathbf{A}(G/H, \mathbf{E}_\chi)$ denote the complex that is the analog of (2.1) with smooth functions replaced by hyperfunctions,

$$(2.2) \qquad \mathbf{A}(G/H, \mathbf{E}_\chi)\colon \quad (B(G) \otimes E_\chi \otimes \Lambda\mathfrak{n}^*)^H, d.$$

The flag variety X (of Borel subalgebras of $\mathfrak{g}$) contains $S = G \cdot \mathfrak{b}$ as a real analytic submanifold. Since H normalizes $\mathfrak{b}$, G/H maps G-equivariantly onto

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