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GROUP ACTIONS AND EQUIVARIANT LIPSCHITZ ANALYSIS

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The idea that one could prove by analytic methods the topological invariance of characteristic classes was suggested in an influential lecture by Singer. The concrete realization of this, in the case of rational Pontrjagin classes, was the result of works by Sullivan, who showed that topological manifolds have essentially unique Lipschitz structures, and Teleman, who showed that Lipschitz manifolds are the type of object for which one can analyze the signature operator. (A refinement of the same method proves the $KO[1/2]$ orientability of topological bundles.)

Much of this can be done for manifolds with group actions.

The main implication is that for certain sorts of topological group actions (including all smooth and PL actions) one can construct an "equivariant signature operator" whose class in equivariant K-homology is a topological invariant. This class satisfies the G-signature theorem. The topological method has implications for various precise classification problems (e.g. semifree topological actions on the sphere), but here we shall present only the consequences of these Lipschitzian ideas:

(A) Topologically locally linear G-manifolds for odd-order G have $KO_G[1/2]$ orientations (Madsen-Rothenberg).

For all finite G there are Lipschitz structures, but the signature class is a zero divisor for even-order groups. The same method produces, upon application of the localization theorem, "Atiyah-Singer" classes for some actions with nonmanifold fixed set (Weinberger, extending earlier joint work with Cappell).

(B) For odd-order groups, topological and linear conjugacy of linear representations are equivalent (Hsiang-Pardon, Madsen-Rothenberg).

This is a consequence of (A). Alternatively one can return to the original argument of Atiyah-Bott-Milnor for semifree representations.

A stronger result is true: the Atiyah-Bott local numbers associated to representations agree for elements that have discrete fixed sets in topologically equivalent representations. This distinguishes many pairs of representations of even-order groups.

(C) $R\,\mathrm{Top}(G) = R\,\mathrm{Lip}(G)$ for all finite G, where $R\,\mathrm{Cat}$ is the Grothendieck group of representations up to Cat conjugacy (Weinberger, after inverting 2).

This implies that there is no analytic definition of Reidemeister torsion that works well in the Lipschitz setting. The reason is that, following

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