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QUANTUM FIELD THEORY IN NINETY MINUTES

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These notes grew out of a ninety-minute lecture delivered in a seminar at the University of Michigan. The audience consisted of mathematicians with a very wide spectrum of research interests. In fact, there was only one other mathematical physicist present. We here attempt to preserve some of the casual flavor of the live seminar. There will be some generalizations, imprecise statements, and disputable implications. These are natural in trying to cover so broad an area as Euclidean quantum field theory, so briefly, demanding no specialized background. On the other hand, we proudly hold up to the experts the accomplishment of here presenting a *complete*, *precise*, *rigorous definition of the two-dimensional quantum field theory* $p(\phi)_2$, *easily accessible to most graduate students in mathematics*. The concepts of cutoffs, renormalization, and perturbation series are touched on, as are some of the features of more complicated theories. Recent theoretical developments have made possible the simplicity and elegance of the present treatment.

Defining a Euclidean quantum field theory (as pioneered by E. Nelson) is exactly the problem of making sense of an initially only formally defined functional integral. We start by listing several example theories in (space-time) dimensions one through four.

I. A particle moving in the potential $V(x)$ (a one-dimensional field theory). Here one integrates over the space whose points are paths,

$$\phi(x)\colon R^1 \to R^1. \tag{1}$$

One should here be impressed with the problem of establishing an integral, or measure, on such a huge, infinite-dimensional space. We put a weighting on the path, $\phi(x)$, given by $e^{-S(\phi)}$,

$$S(\phi) = \int \left[\frac{1}{2}\left(\frac{d\phi}{dx}\right)^2 + V(\phi)\right] dx. \tag{2}$$

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