BOOK REVIEWS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 16, Number 2, April 1987

0273-0979/87 $1.00 + $.25 per page

Treatise on the shift operator. Spectral function theory, by N. K. Nikol'skiĭ, with an appendix by S. V. Hruščev and V. V. Peller. Translated from the Russian by Jaak Peetre. Grundlehren der mathematischen Wissenschaften, vol. 237, Springer-Verlag, Berlin, Heidelberg, New York, and Tokyo, 1986, $64.50. ISBN 0-387-15021-8

The shift (or unilateral shift) operator, in one of its two common guises, is the operator on the complex Hilbert space l^2 that sends the sequence $(c_0, c_1, c_2, \ldots)$ to the sequence $(0, c_0, c_1, \ldots)$. It is a typically infinite-dimensional operator and as such is often useful to the instructor of an introductory operator theory course as a handy concrete illustration of various phenomena–for instance, it is left invertible but not right invertible; it has no eigenvalues, yet the eigenvalues of its adjoint fill the open unit disk; the powers of its adjoint tend to 0 in the strong operator topology, yet its powers do not.

In its other common guise, the shift operator is the operator of "multiplication by z" on the Hardy space H^2, the space of holomorphic functions in the unit disk whose power series coefficients at the origin are square summable. That guise is the appropriate one to visualize when one investigates the structure of the shift operator more deeply, a point strikingly brought home by A. Beurling in 1949, when he showed that the invariant subspace structure of the shift operator mimics the factorization theory in H^2 (the inner-outer factorization). Since Beurling's pioneering work, it has been recognized that this innocent-looking operator is connected with a surprisingly vast body of function theory.

The adjoint of the direct sum of countably many copies of the shift operator, the so-called backward shift operator of multiplicity $\aleph_0$, has a remarkable universality property discovered by G.-C. Rota: every operator on a separable Hilbert space whose spectrum is contained in the open unit disk is similar to a part of that operator (that is, similar to the restriction of that operator to one of its invariant subspaces). From this and related results one sees that, in principle, complete knowledge of the shift operator of multiplicity $\aleph_0$ would entail complete knowledge of all Hilbert space operators. In a less fanciful vein, one can hope on the basis of such results that by better understanding the shift operators of various multiplicities one will gain better insight into the structures of other operators and possibly of operators in general. The pursuit of that goal is a major ongoing program in operator theory.