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BRAIDS, HYPERGEOMETRIC FUNCTIONS, AND LATTICES

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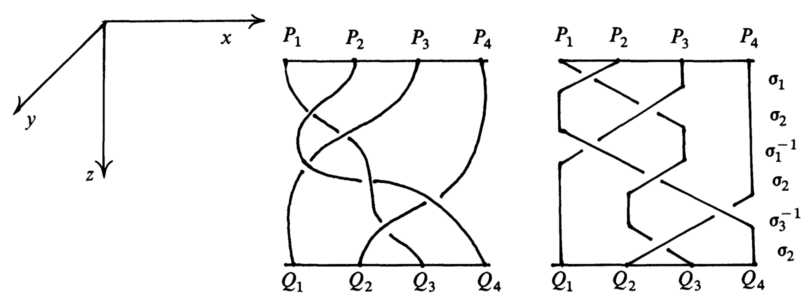
1. Braids. Let L_1, L_2 be two parallel lines in the plane $y = 0$ of (x, y, z) space, L_1 at $z = r_1$ and L_2 at $z = r_2$. Let $P_i = (i, 0, r_1)$, $Q_i = (i, 0, r_2)$, $i = 1, \ldots, n$.

A *braided n-path* is a set of n paths $c_i(t)$ in $\mathbf{R}^3$ $(i = 1, \ldots, n)$ satisfying

(1) $c_i(t) = (x_i(t), y_i(t), t)$, $r_1 \leqslant t \leqslant r_2$, $c_i(r_1) = P_i$, $c_i(r_2) \in \{Q_1, \ldots, Q_n\}$.

(2) The paths do not intersect.

Two braided n-paths are regarded as *equivalent* if and only if it is possible to deform the one configuration into the other respecting conditions (1) and (2) throughout the deformation; note that one does permit r_1, r_2 to vary so long as $r_1 < r_2$ is respected. Thus (a) and (b) in Figure 1 represent the same braid. By definition, a *braid* is an equivalence class of braided n-paths.



(a) (b) $\sigma_2\sigma_3^{-1}\sigma_2\sigma_1^{-1}\sigma_2\sigma_1$

FIGURE 1.

Two braids A and B can be multiplied: $B \cdot A$ is the braid obtained by first braiding A then B, and adjusting the domain of the parameter t so that it changes without interruption, i.e., by bringing the end line of A and initial line of B together and then erasing them.

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