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INDEX THEORY FOR TOEPLITZ OPERATORS ON BOUNDED SYMMETRIC DOMAINS

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In this note we give an index theory for Toeplitz operators on the Hardy space of the Shilov boundary of an arbitrary bounded symmetric domain. Our results generalize earlier work of Gohberg-Krein and Venugopalkrishna [**12**] for domains of rank 1 and of Berger-Coburn-Korányi [**1**] for domains of rank 2.

Bounded symmetric domains (Cartan domains, classical or exceptional) are the natural higher-dimensional analogues of the open unit disk. Each such domain is homogeneous under a semisimple Lie group of biholomorphic transformations and has an (essentially unique) realization as a convex circular domain which is conveniently described in Jordan algebraic terms: The underlying vector space $Z \approx \mathbf{C}^n$ carries a *Jordan triple product*, denoted by $(u, v, w) \mapsto \{uv^*w\}$, and the bounded symmetric domain D is the open unit ball of Z with respect to the "spectral norm" [**7**]. The basic example is the space $Z = \mathbf{C}^{p\times q}$ of rectangular matrices, with Jordan triple product $\{uv^*w\} := (uv^*w + wv^*u)/2$, giving rise to the "hyperbolic matrix ball" $D = \{z \in Z\colon \text{ spectrum } (z^*z) < 1\}$.

The symmetric domains of rank 1 are the (smooth) Hilbert balls in $\mathbf{C}^n$. For domains D of higher rank r, the boundary ∂D forms a nonsmooth "stratified space". In Jordan algebraic terms, the structure of ∂D can be described as follows: An element $e \in Z$ satisfying $\{ee^*e\} = e$ is called a *tripotent*. For matrices, the tripotents are just the partial isometries. Every tripotent e induces a splitting of Z into the "Peirce spaces" $Z_\lambda(e) := \{z \in Z\colon \{ee^*z\} = \lambda z\}$ for $\lambda = 0, \frac{1}{2}, 1$. The set

$$D^e := D \cap Z_0(e) \tag{1}$$

is a bounded symmetric domain in $Z_0(e)$ and, by [**7**], the translated sets $e + D^e$, for nonzero tripotents e, constitute all *boundary components* ("faces") of D. For $j \leq r$, these components can be organized into smooth families, labeled by the *compact manifold* S_j *of all tripotents of equal rank* j. The Shilov boundary S of D coincides with S_r [**7**]. S is homogeneous under the connected linear automorphism group K of D and thus carries a K-invariant probability measure. For irreducible D, the same is true of the "partial Shilov boundaries" S_j. We assume in the following that D is irreducible.

The *Hardy space* $H^2(S)$ over S and the dense subspace $\mathcal{P}(Z)$ of all polynomials have been analyzed by W. Schmid [**8**], who showed that $\mathcal{P}(Z)$ has an iso-

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