BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 16, Number 1, January 1987

SYMPLECTIC GROUPOIDS AND POISSON MANIFOLDS

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0. Introduction. A symplectic groupoid is a manifold Γ with a partially defined multiplication (satisfying certain axioms) and a compatible symplectic structure. The identity elements in Γ turn out to form a Poisson manifold Γ_0, and the correspondence between symplectic groupoids and Poisson manifolds is a natural extension of the one between Lie groups and Lie algebras.

As with Lie groups, under certain (simple) connectivity assumptions, every homomorphism of symplectic groupoids is determined by its underlying Poisson mapping, and every Poisson mapping may be integrated to a canonical relation between symplectic groupoids. On the other hand, not every Poisson manifold arises from a symplectic groupoid, at least if we restrict our attention to ordinary manifolds (even non-Hausdorff ones), so "Lie's third fundamental theorem" does not apply in this context.

Using the notion of symplectic groupoid, we can answer many of the questions raised by Karasev and Maslov [**9**, **10**] about "universal enveloping algebras" for quasiclassical approximations to nonlinear commutation relations. (I wish to acknowledge here that [**9**] already contains implicitly some of the ideas concerning Poisson structures and their symplectic realizations which were presented in [**18**].) In fact, the reading of Karasev and Maslov's papers was one of the main stimuli for the work described here. Following their reasoning, it seems that a suitably developed "quantization theory" for symplectic groupoids should provide a tool for studying nonlinear commutation relations which is analogous to the use of topology and analysis on global Lie groups in the study of *linear* commutation relations. Such a theory would also clarify the relation, mostly an analogy at present, between symplectic groupoids, star products [**2**], and the operator algebras of noncommutative differential geometry [**3**].

More immediately, the notion of symplectic groupoid unifies many constructions in symplectic and Poisson geometry; in particular, it provides a framework for studying the collection of all symplectic realizations of a given Poisson manifold.

A detailed exposition of these results will appear in [**4**]. Many of the details were worked out during a visit to the Université Claude-Bernard Lyon I. I would like to thank Pierre Dazord for his hospitality in Lyon, as well as for many stimulating discussions. The idea of introducing groupoids into symplectic geometry arose in the course of conversations with Marc Rieffel about operator algebras and the subsequent reading of J. Renault's thesis [**14**].

Received by the editors July 28, 1986.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 58F05; Secondary 20L15.