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FATOU THEOREMS ON DOMAINS IN $\mathbf{C}^n$

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If $D \subseteq \mathbf{C}$ is the unit disc and $0 < p < \infty$ then define $H^p(D)$ to be those f holomorphic on D such that

$$||f||_{H^p} \equiv \sup_{0<r<1} \int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta^{1/p} < \infty.$$

The space $H^\infty(D)$ consists of the bounded holomorphic functions equipped with the supremum norm. The classical Fatou theorem asserts that if $f \in H^p(D)$ then for a.e. (with respect to linear measure in ∂D) $e^{i\theta} \in \partial D$ it holds that

$$\lim_{r \to 1^-} f(re^{i\theta}) \equiv f^*(e^{i\theta})$$

exists. For $p \geq 1$, the function f can be recovered from f^* by means of the Cauchy or Poisson integral formulas. See [**JG** or **SK**].

It is an important and useful fact that this radial approach to $e^{i\theta} \in \partial D$ may be replaced by a more general type of approach: if $\alpha > 1$ and $P \in \partial D$ we define the *Stolz region*

$$\Gamma_\alpha(P) = \{z \in D \colon |z - P| < \alpha \cdot (1 - |z|)\}.$$

Then, for any $0 < p \leq \infty$, $f \in H^p(D)$, and $\alpha > 1$ we have

$$\lim_{\Gamma_\alpha(P) \ni z \to P} f(z) = f^*(P)$$

for a.e. $P \in \partial D$. It is known [**IP**] that this nontangential method of approach is best possible.

If $\Omega \subseteq \mathbf{C}^n$ is a smoothly bounded domain then there is a similar theory of H^p spaces (also classical). (Let $\delta_\Omega(z) \equiv \operatorname{dist}(z, \partial\Omega)$.) In this theory one replaces

(i) the circles $\{re^{i\theta}\colon 0 \leq \theta < 2\pi\}$ by $\partial\Omega_\varepsilon \equiv \{z \in \Omega\colon \delta_\Omega(z) = \varepsilon\}$, ε small;
(ii) linear measure by $(2n-1)$-dimensional area measure;
(iii) Stolz regions Γ_α by cones in space of fixed aperture.

It is a remarkable discovery of Korányi (for the ball and for certain symmetric domains [**AK1**, **AK2**]) and of Stein (for smoothly bounded domains [**ES1**]) that in $\mathbf{C}^n$, $n > 1$, the conical approach regions are not optimal for studying the boundary behavior of H^p functions. Indeed, they may be replaced by ***admissible approach regions*** which are conical in "complex normal directions" and parabolic in "complex tangential directions" (see [**ES1**, **SK**]).

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