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Theory of multipliers in spaces of differentiable functions, by V. G. Maz'ya and T. O. Shaposhnikova, Monographs and Studies in Mathematics, vol. 23, Pitman Publishing Co., Brooklyn, New York, 1985, xii + 344 pp., $49.95. ISBN 0-273-08638-3

1. Multipliers. One of the simplest examples of a multiplier in a space of differentiable functions is a measurable function $\gamma(x)$, $x \in \mathbf{R}^n$, such that the operator of pointwise multiplication $u \to \gamma \cdot u$ is bounded from the Sobolev space W_2^1 on $\mathbf{R}^n$ into L_2 on $\mathbf{R}^n$; equivalently, there is a constant c such that

$$(1) \qquad \int |\gamma(x) \cdot \phi(x)|^2 \, dx \leqslant c \int \left(|\nabla \phi(x)|^2 + |\phi(x)|^2 \right) dx$$

for all $\phi \in C_0^\infty(\mathbf{R}^n)$. The space of all such γ is denoted by $M(W_2^1 \to L_2)$, with the smallest c in (1) the square of the multiplier norm of γ. Clearly, one can easily extend this notion to pairs of higher-order Sobolev spaces: $W_p^m \to W_q^k$, $k \leqslant m$, $1 \leqslant p, q < \infty$, or for that matter, to any of the various pairs of function spaces that naturally occur in analysis. The coefficients of a differential operator acting on Sobolev functions can be interpreted as multipliers. For example, if $P(x, D)u = \sum_{|\alpha| \leqslant k} a_\alpha(x) D_x^\alpha u$, then P: $W_p^m \to W_p^{m-k}$ is continuous when $a_\alpha \in M(W_p^{m-|\alpha|} \to W_p^{m-k})$. The function γ is called a compact multiplier if the operator of pointwise multiplication is a compact operator. The principal theme of the book under review (referred to below as *Multipliers*) is the characterization of multipliers and compact multipliers in the basic Sobolev-type spaces used in analysis. Because of their connection to differential equations, it is not surprising that there are plenty of sufficient conditions in the literature for multipliers or compact multipliers. For example,