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Linear turning point theory, by Wolfgang Wasow, Applied Mathematical Sciences, vol. 54, Springer-Verlag, New York, Berlin, Heidelberg and Tokyo, 1985, ix + 246 pp., $38.00. ISBN 0-387-96046-5

The transitions of solutions of a linear differential equation from oscillatory to exponentially growing or exponentially decaying behavior as the independent variable, for example, changes sign are phenomena of interest to physicists and other scientists, primarily in the past sixty years, and continuing even today. The simplest equation exhibiting such behavior is Airy's equation

$$y'' + xy = 0, \tag{A}$$

obtained from his study of the rainbow [**2**]. If we set $x = [\psi(0)\varepsilon^{-2}]^{1/3}t$, then the rainbow equation (A) becomes

$$\varepsilon^2 d^2y/dt^2 + t\psi(0)y = 0, \tag{A*}$$

which might well be expected to have solutions close to solutions of the equation

$$\varepsilon^2 d^2y/dt^2 + t\psi(t)y = 0 \qquad (\psi(0) \neq 0). \tag{A\#}$$

This idea occured to R. Gans in 1915 [**6**] in his investigations of total reflection in physical—as opposed to geometrical—optics. The point $t = 0$ is called a (simple) turning point, one where solutions of (A#) change from oscillatory to exponential behavior. An obvious mathematical question, only answered much later by R. E. Langer [**8, 9**] and others, is whether one can find changes of variables in (A) such that (A#) becomes (A*) with $\psi(0) = 1$ and with a small error term included. If the transformation from (A#) to A(*) is exact, it turns out that often it is but a formal power series in ε with coefficients holomorphic in the complex variable x, i.e., an asymptotic series which converges only for