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THE CYCLIC HOMOLOGY AND K-THEORY OF CURVES

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ABSTRACT. It is now possible to calculate the K-theory of a large class of singular curves over fields of characteristic zero. Roughly speaking, the K-theory of a curve is the K-theory of its (smooth) normalization plus a few shifted copies of the K-theory of the field plus a "nil part." The nil part is a vector space depending only on the analytic type of the singularities, and may be computed locally. We completely compute the nil part for seminormal curves and give a conjectural calculation in general which depends upon cyclic homology.

Until recently, very little has been known about the higher algebraic K-theory of anything but finite fields. In this note we announce the computability of the K-theory of singular curves in characteristic zero in terms of the K-theory of smooth curves and fields. If the curve is seminormal, we give a complete calculation; otherwise, the calculation depends on the validity of:

CONJECTURE. *Let B be a finite integral extension of a ring A, and let I be the conductor ideal. Assume A contains $\mathbf{Q}$, the rational numbers. Then the map*

$$K_*(A,B,I) \to HC_{*-1}(A,B,I)$$

is an isomorphism, where the right-hand term is double relative cyclic homology taken over the field $\mathbf{Q}$.

This conjecture is known when $B = A/J$ [**OW**]. In the absence of this conjecture, all our results can be interpreted as calculations of the cyclic homology of affine curves. In order to more simply present our results, let us set

$$V_n = \begin{cases} 0 & n = 0, 1, \\ k \oplus \Omega_k^2 \oplus \Omega_k^4 \oplus \cdots \oplus \Omega_k^{n-2} & n \text{ even}, n \geq 2, \\ \Omega_k \oplus \Omega_k^3 \oplus \cdots \oplus \Omega_k^{n-2} & n \text{ odd}, n \geq 3. \end{cases}$$

Here Ω_k^i denotes the ith exterior of the module Ω_k of Kähler differentials of k over $\mathbf{Q}$. As an illustration, we present

CURVE 1 (TWO INTERSECTING LINES). Let k be a field of characteristic zero, and set $A = k[x,y]/(xy)$, $I = (x,y)A$, and $X = \mathrm{Proj}(k[X,Y,Z]/(XY))$.

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