RESEARCH ANNOUNCEMENTS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 15, Number 2, October 1986

A WEAK RAMANUJAN CONJECTURE FOR GENERIC CUSPIDAL SPECTRUM OF QUASI-SPLIT GROUPS

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The strong form of the Ramanujan conjecture for a quasi-split reductive group had predicted that all the components of a cusp form are tempered (cf. [**5**, **13**]). But, examples of Kurokawa (cf. [**11**]) and Howe and Piatetski-Shapiro [**5**] have shown that this is not true in general. In fact, even for PSP_4 there are cusp forms which already defy the conjecture. Consequently, in [**11**] Langlands predicted that an automorphic representation fails to be tempered only if it lifts to an anomalous representation of some $GL(n)$.

On a quasi-split group, one may consider the class of cusp forms which as representations can be realized on spaces of functions which transform on the left according to a generic character of the unipotent radical of a Borel subgroup (i.e., the ones with Whittaker models). We call such automorphic representations *generic*. None of the nontempered automorphic representations constructed so far are generic. In what follows, we shall produce some evidence towards the validity of the strong Ramanujan conjecture for generic automorphic forms (Theorems 1 and 2, and Corollary 2). In fact, in Corollary 2, we obtain a uniform bound for the Hecke eigenvalues of generic cusp forms on many absolutely simple quasi-split groups over number fields. It also provides us with a new proof of the best available estimate for the Fourier coefficients of Maass wave forms (Corollary 4). Detailed proofs will appear elsewhere.

Let $\mathbf{G}$ be a quasi-split group over a number field F. Set $G = \mathbf{G}(\mathbf{A}_F)$. Let $\mathbf{P}$ be a maximal F-parabolic subgroup of $\mathbf{G}$. Write $\mathbf{P} = \mathbf{MN}$ and let P, M, and N be the corresponding groups of adelic points. Let v be a place of F and denote by G_v, P_v, M_v, and N_v the corresponding local groups of F_v-rational points. Let σ be a cusp form on M and write $\sigma = \bigotimes_v \sigma_v$, a restricted tensor product of representations of the local groups G_v (cf. [**3**]).

Received by the editors May 31, 1986.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 10D40, 12A70, 22E55; Secondary 22E35.
Partially supported by NSF grant MCS-8320317.