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FOLIATIONS AND SURGERY ON KNOTS

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In this note we will first discuss some of the properties of 3-manifolds which possess taut (defined below) foliations. Next we will describe our main results which assert that many surgered 3-manifolds possess taut foliations. Finally we will show how these existence theorems together with the previously stated results can be exploited to produce topological corollaries, e.g., knots in $S^2 \times S^1$ not contained in 3-cells are determined by their complements, and knots in S^3 have property R.

For clarity and brevity many of the results are not stated in full generality and the discussion of smooth versions of the announced foliations results is omitted. Details to all the results can be found in [**G2**, **G3**, and **G4**].

Let $\mathcal{F}$ be a transversely oriented codimension one foliation on a compact oriented 3-manifold M such that $\mathcal{F}$ is transverse to ∂M. $\mathcal{F}$ is *taut* if for each leaf L of $\mathcal{F}$ there exists a curve γ transverse to $\mathcal{F}$ such that $\gamma \cap L \neq \emptyset$.

The existence of such a taut $\mathcal{F}$ implies that ∂M is a (possibly empty) union of tori and M is either $S^2 \times S^1$ (and $\mathcal{F}$ is the product foliation) or M is irreducible. The first condition follows by Euler characteristic reasons, while the latter follows from the work of Reeb [**R**], Haefliger, Novikov [**N**], and Alexander [**A**]. See also [**Ro**]. [Historical note: Armed with the work of [**R** and **N**] the reader will see in [**A**] a proof of the fact that a separating 2-sphere in a tautly foliated 3-manifold bounds a 3-cell. This proof was rediscovered in [**Ro**]]. Novikov also showed that a curve transverse to $\mathcal{F}$ is homotopically of infinite order. Thurston [**T**] has shown that any compact leaf L is a Thurston norm-minimizing surface [i.e., $|\chi(L')| \leq |\chi(T')|$ for any properly embedded T with $[T] = [L] \in H_2(M, \partial M)$, where S' denotes $S-$(sphere and disc components)] for the class $[L] \in H_2(M, \partial M)$. Geometrically (if $\mathcal{F}$ is also C^2) Sullivan [**S**] has shown that M possesses a Riemannian metric such that each leaf is minimal, i.e., locally area minimizing. In fact by Harvey and Lawson $\mathcal{F}$ can be calibrated [**HL**].

DEFINITIONS. Let M be a 3-manifold such that ∂M contains a torus T. N is said to be obtained by *Dehn filling* M along an essential simple closed curve γ in T, if N is obtained by first attaching a 2-handle to M along γ and then capping off the resulting 2-sphere with a 3-cell. N is obtained by attaching a solid torus, called the *filling*, to M and $M = N - \mathring{N}(k)$ where k is the *core* of the filling. Note that a manifold obtained by Dehn surgery on a knot $k \subset N$ is a manifold obtained by Dehn filling $N - \mathring{N}(k)$. A manifold

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