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THE CLASSIFICATION OF MAPS OF SURFACES

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In this note we discuss the topology of maps of positive degree between closed orientable surfaces. Two maps $f, g: M \to N$ are said to be *equivalent* if there exist homeomorphisms $h: M \to M$ and $k: N \to N$ such that $k \circ f = g \circ h$ (or $k \circ f \simeq g \circ h$ in the homotopy category). If k is homotopic to id_N we say f and g are *strongly equivalent*. The notion of equivalence is analogous to a change of basis in domain and range in linear algebra.

Surface maps of special interest are branched coverings, i.e., $f: M \to N$ is a *branched covering* if there exists a finite set of points $B \subset N$ such that $f|M - f^{-1}(B)$ is a covering map. An arbitrary branched covering may be approximated by a *generic branched covering*, i.e., one in which each point of N has degree (f) or degree $(f) - 1$ preimages.

One of the first people to study branched coverings was Riemann, who proved in his thesis (1851) that Riemann surfaces occur as conformal branched coverings of S^2. In 1871 and 1873 the classical function theorists Lüroth and Clebsch succeeded in showing that generic branched coverings of S^2 are classified up to (strong) equivalence by their degree. The classification problem for general range N was reduced by Hurwitz in 1891 to the algebraic-combinatorial study of representations of $\pi_1(N - B)$ into Σ_d, the symmetric group on d letters where $d =$ degree of the branched covering.

In 1928 Reidmeister showed that there is a 1-1 correspondence betweeen subgroups of $\pi_1(N)$ and covering spaces of N. This allows a generic branched covering $\phi: M \to N$ to be factored uniquely as a primitive (surjective on π_1) generic branched covering $\tilde{\phi}: M \to \tilde{N}$ followed by an unbranched covering map $p: \tilde{N} \to N$ corresponding to the image of ϕ on π_1.

Primitive generic branched coverings were shown to be classified by their degree by Hamilton in 1966 for arbitrary N provided that $b \geq 2d$, where b is the number of branch points and d is the degree. This was improved by Berstein and Edmonds in 1979 and 1984 to $b > d/2$ and arbitrary N, or with no restriction on b to $N = S^1 \times S^1$. More importantly, Berstein and Edmonds stressed that primitive generic branched coverings should be classified up to equivalence by their degree and they conjectured a suggestive normal form.

Recently we have shown that primitive generic branched coverings are actually classified up to strong equivalence by their degree, and consequently we prove the following theorem.

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