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ALGEBRAIC K-THEORY OF HYPERBOLIC MANIFOLDS

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ABSTRACT. Let $\Gamma = \pi_1 M$ where M is a complete hyperbolic manifold with finite volume. We announce (among other results) that $\mathrm{Wh}\,\Gamma = 0$ where $\mathrm{Wh}\,\Gamma$ is the Whitehead group of Γ. We also announce $\mathrm{Wh}_2\,\Gamma = 0$, $\tilde{K}_0(\mathbf{Z}\Gamma) = 0$, $K_{-n}(\mathbf{Z}\Gamma) = 0$ (for $n > 0$), and $\mathrm{Wh}_n\,\Gamma \otimes \mathbf{Q} = 0$ (for all n). We calculate the weak homotopy type of the stable topological concordance space $\mathfrak{C}(M)$, and hence Waldhausen's Wh^{PL}-theory (cf. [**22**]) of M, in terms of simpler stable concordance spaces. When M is compact, the calculation is in terms of $\mathfrak{C}(S^1)$ where S^1 is the circle.

A connected complete Riemannian manifold M is called *weakly admissible* if there exist positive real numbers $a < b$ such that all the sectional curvatures of M are less than $-a$ and bigger than $-b$. A weakly admissible manifold is *admissible* if it has finite volume. In particular, all complete locally symmetric spaces having finite volume and strictly negative sectional curvatures are admissible Riemannian manifolds. These are precisely the real, complex, quaternionic and Cayley complete hyperbolic manifolds of finite volume. All complete manifolds of constant negative sectional curvature and finite volume occur among these; in fact, they are the complete real hyperbolic manifolds of finite volume. The purpose of this paper is to announce the calculation of the algebraic K-theory of admissible manifolds.

We start by stating that the Whitehead group $\mathrm{Wh}\,\pi_1 M$ of the fundamental group of an admissible manifold M vanishes. Actually, we proceed to formulate and state a bit more general result. A group Γ is *K-flat* if $\mathrm{Wh}(\Gamma \oplus C^n) = 0$ for all nonnegative integers n where C^n denotes the free abelian group of rank n. The Bass-Heller-Swan formula [**3**] implies $\mathrm{Wh}\,\Gamma = 0$, $\tilde{K}_0(\mathbf{Z}\Gamma) = 0$ and $K_{-n}(\mathbf{Z}\Gamma) = 0$ provided Γ is K-flat and $n > 0$.

A smooth fiber bundle $F \to E \underset{p}{\to} M$ is *admissible* if

(i) M is admissible;

(ii) F is a closed connected manifold;

(iii) for each virtually poly-$\mathbf{Z}$ subgroup S of $\pi_1 M$, $p_{\#}^{-1}(S)$ is a K-flat subgroup of $\pi_1 E$.

A group Γ is admissible if it is isomorphic to the fundamental group of the total space E of an admissible fiber bundle. The main result of [**9**] is: Any torsion-free virtually poly-$\mathbf{Z}$ group is K-flat. Consequently, the fundamental group of an admissible Riemannian manifold is admissible. In particular, any torsion-free discrete subgroup Γ of the Lie group G, where G is either $O(1,n)$,

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