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ASYMPTOTIC COMPLETENESS OF SHORT-RANGE MANY-BODY SYSTEMS[1]

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ABSTRACT. We announce a proof of asymptotic completeness for quantum mechanical systems consisting of arbitrary numbers of particles interacting via short-range forces (see conditions (A)–(D) below). A complete proof is given in [**SigSof 1**]. Previously, there were only partial results in this direction (see reviews and discussions in [**Enss, RSIII, Sig**]).

Consider an N-body system in the physical space $\mathbf{R}^\nu$. The configuration space in the center-of-mass frame is $X = \{x \in \mathbf{R}^{\nu N} | \sum m_i x_i = 0\}$ with the inner product $\langle x, y\rangle = 2\sum m_i x_i \cdot y_i$. Here $m_i > 0$ are masses of the particles in question. The *Schrödinger operator* of such a system is

$$H = -\Delta + V(x) \quad \text{on } L^2(X).$$

Here Δ is the Laplacian on X and $V(x) = \sum V_{ij}(x_i - x_j)$. We assume that the potentials V_{ij} are real and obey (with $\langle x\rangle = (1 + |x|^2)^{1/2}$)

(A) $V_{ij}(y)$ are Δ_y-compact,
(B) $\langle y\rangle^{1+\theta}|\nabla V_{ij}(y)|$ are Δ_y-bounded for some $\theta > 0$,
(C) $|y|^2 \Delta V_{ij}(y)$ are Δ_y-bounded,
(D) $\langle y\rangle^\mu V_{ij}(y)$ are Δ_y-bounded.

Due to condition (A), H is selfadjoint on $D(H) = D(\Delta)$ by the Kato theorem (see e.g. [**RSII**]). By a *short-range system* we understand a system obeying (A) and (D) with $\mu > 1$. Our main result is

THEOREM 1. *Assume an N-body system is described by potentials obeying conditions* (A)–(D) *with $\mu > 1$. Then asymptotic completeness holds for this system.*

Conditions (B)–(D) can be relaxed if we exercise more care in our estimation.

Denote by $a, b, \ldots$, *partitions* of the set $\{1, \ldots, N\}$ into nonempty disjoint subsets, called clusters. The relation $b \subset a$ means that b is a refinement of a. We assume that partitions have at least two clusters. We define the *intercluster interaction* for a partition a as

$$I_a = \text{sum of all potentials linking different clusters in } a,$$

and the Hamiltonian for the system composed of the *noninteracting clusters*: $H_a = H - I_a$. Furthermore, H^a stands for the Hamiltonian of the same

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