RESEARCH ANNOUNCEMENTS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 14, Number 1, January 1986

DEFORMATION SPACES ASSOCIATED TO COMPACT HYPERBOLIC MANIFOLDS

BY DENNIS JOHNSON AND JOHN J. MILLSON

Recently, there has been considerable interest in spaces of locally homogeneous (or geometric) structures on smooth manifolds, motivated by Thurston [**6**, **7**]. If M is a smooth manifold, we will let $\mathcal{C}(M)$ denote the space of conformal structures (with marking) on M and $\mathcal{P}(M)$ the space of projective structures (with marking) on M. Since these spaces are a measure of the complexity of the fundamental group, it makes sense to consider the case in which M admits a hyperbolic structure. We note that in case n, the dimension of M, is strictly greater than 2, this hyperbolic structure is unique by the Mostow Rigidity Theorem. Hence, $\mathcal{C}(M)$ and $\mathcal{P}(M)$ each have a finite number of distinguished points, the conformal and projective structures associated to the hyperbolic structure with the various possible markings of $\pi_1(M)$.

In order to study $\mathcal{C}(M)$ and $\mathcal{P}(M)$, it is convenient to replace $\mathcal{C}(M)$ and $\mathcal{P}(M)$ with the space of conjugacy classes of representations of Γ, the fundamental group of M, into the automorphism groups $SO(n+1,1)$ and $PGL_{n+1}(\mathbf{R})$ of the model spaces S^n and $\mathbf{RP}^n$. This is possible because of a general result of Lok [**2**].

Let $\mathcal{S}(M)$ be a space of (marked) locally homogeneous structures modelled on a homogeneous space $X = G/H$ with G a semisimple linear algebraic group. Given a structure $s \in \mathcal{S}(M)$, by continuing coordinate charts around elements of Γ, we obtain the holonomy representation ρ of Γ into G and a map

$$\text{hol}\colon \mathcal{S}(M) \to \text{Hom}(\Gamma, G)/G$$

defined so that $\text{hol}(s)$ is the orbit of ρ under conjugation by G. Then Theorem 1.11 of Lok [**2**] states that hol is an open map which lifts to a local homeomorphism from the space of (G,X)-developments to $\text{Hom}(\Gamma, G)$. We will refer to this result as the "Holonomy Theorem". Unfortunately hol is not necessarily a local homeomorphism. To deal with this point we say that a representation ρ of Γ is *stable* if the image of ρ is not contained in a parabolic subgroup

Received by the editors September 20, 1984 and, in revised form, July 29, 1985.
1980 *Mathematics Subject Classification*. Primary 22E40.