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ON FINITENESS OF THE NUMBER OF STABLE MINIMAL HYPERSURFACES WITH A FIXED BOUNDARY

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ABSTRACT. Can there be infinitely many minimal hypersurfaces with a given boundary in a Riemannian manifold? A number of previous results, positive and negative, already indicated that the answer depends on the definition of surface, on orientability, on stability and minimizing properties of the surface, on the smoothness and geometry of the boundary, and on the ambient manifold.

1. Finiteness for area-minimizing hypersurfaces in certain manifolds. Several years ago, R. Hardt and L. Simon [**HS**, 1979, §12], following the original work of F. Tomi [**T2**, 1974], proved that a $C^{4,\alpha}$ Jordan curve Γ in $\mathbf{R}^3$ bounds only finitely many orientable, area-minimizing immersed manifolds-with-boundary of least area. The immersed manifolds are assumed to minimize area over all immersed manifolds of finite topological type. It has been more or less understood that this finiteness theorem applies as well to hypersurfaces in $\mathbf{R}^n$ ($n \leq 7$), bounded by a compact, connected, $C^{4,\alpha}$ submanifold B of $\mathbf{R}^n$.

Such finiteness for oriented area-minimizing hypersurfaces fails in general ambient manifolds. For example, in the standard 2-dimensional sphere, there is a continuum of length-minimizing paths from the south pole to the north pole. However, we prove the following finiteness theorem for a *noncompact, real-analytic* Riemannian manifold.

THEOREM A. *Fix $n \leq 7$, $0 < \alpha < 1$. Let N be an n-dimensional, complete, connected, noncompact, real-analytic Riemannian manifold, with sectional curvature bounded above and injectivity radius bounded away from 0. Let B be a compact, oriented $(n-2)$-dimensional, $C^{2,\alpha}$ submanifold of N, with a positive integer multiplicity assigned to each component. Let $\mathcal{M}$ be the space of equivalence classes (under reparameterization) of $C^{2,\alpha}$ immersions f of compact, orientable, $(n-1)$-dimensional $C^{2,\alpha}$ manifolds-with-boundary M into N, such that $f \mid \partial M$ is a covering of B with the prescribed multiplicities. Let $a = \inf\{\text{area}\, f \colon [f, M] \in \mathcal{M}\}$. Then $\mathcal{M}_0 = \{[f, M] \in \mathcal{M} \colon \text{area}\, f = a\}$ is finite.*

When the ambient manifold N is compact, it is shown that infinitely many area-minimizing hypersurfaces with a common, smooth boundary necessarily give an "open book" decomposition of N (cf. [**Wi**, **G**]). It follows that if $N = \mathbf{C}P^2$, for example, or if N is nonorientable, there are only finitely many area-minimizing hypersurfaces with a common, smooth boundary.

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