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Random fields: analysis and synthesis, by Erik Vanmarcke, MIT Press, Cambridge, Mass., 1983, xii + 382 pp., $45.00. ISBN 0-262-22026-1

Spectral theory of random fields, by M. I. Yadrenko, Optimization Software, Inc., New York, N.Y., 1983, iii + 259 pp., $24.00. ISBN 0-911575-00-6

A stochastic process is a family of random variables (X_t) defined on some probability space. The parameter t assumes values in an index set T, and X_t assumes values in a measure space called the state space. Much of the theory developed originally in the cases where T is the set of positive integers or where T is the nonnegative real axis. In this context we think of X_t as a random function of time. While this theory was sufficient for the modeling of random functions measured over time, there arose the need for the consideration of random functions X_t where t assumes values in subsets of the plane or space. For example, weather variables recorded at various points in the atmosphere may be viewed as the realization of a stochastic process with $T = R^3$.

A *random field* is simply a stochastic process with T a measurable subset of R^d, for some $d \geqslant 1$. The pioneering works in the theory of stochastic processes of Lévy, Kolmogorov, Khintchine, Feller, and Doob were concerned almost exclusively with the case $d = 1$. But it was Lévy who first noticed the potential interest in the case $d \geqslant 1$. His early work on the Brownian motion of several parameters **[8]**, now know as "Lévy's Brownian motion", pointed out the direction for much modern research in random fields. In recent years interest in random fields has penetrated the theory of martingales, Markov processes, Gaussian processes, additive processes, and general second-order stationary processes.

The branch of probability theory commonly called random fields, in particular, the material considered in the two books under review, has a scope more limited than that suggested by previous remarks. It is the subset of the theory dealing with extensions from $d = 1$ to $d \geqslant 1$ of the theory of processes that are stationary or have stationary increments, either in the strict sense or in the wide sense (second-order stationarity). Another recent book in this area is *The geometry of random fields* by R. J. Adler **[1]**.

Second order random fields and isotropy. As is well known, (X_t), $t \in R^1$, is called second-order (wide sense) stationary if $EX_t = 0$ (or any constant) and $EX_s X_{t+s} = r(t)$, where the latter does not depend on s. If R^1 is extended to R^d, s and t belong to R^d, and if the condition stated above holds, then the random field is called homogeneous. The classical spectral theory in R^1 extends directly to R^d. This extension assumes significant interest when the assumption of *isotropy* is introduced: X_t is isotropic if the function $r(t)$ above depends on t only through the nonnegative variable $\|t\|$, where $\|\cdot\|$ is the usual R^d norm. The