BOOK REVIEWS

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Sequences and series in Banach spaces, by Joseph Diestel, Graduate Texts in Mathematics, Springer-Verlag, New York, Berlin, Heidelberg, Tokyo, 1984, xi + 261 pp., $38.00. ISBN 0-387-90859-5

Many of the problems that motivated the development of functional analysis had to do with sequences and series in various linear spaces. As outstanding examples, consider the expansion of a "general" function in terms of a given sequence of functions (Fourier series or, more generally, the expansion with respect to eigenfunctions of integral or differential operators), or the different convergence notions of sequences of measurable functions (almost everywhere, in measure, in L^p for some $1 \leqslant p \leqslant \infty$), or the convergence of certain sequences of operators (e.g., convolutions with suitable approximate identities).

The results sought were either of a general nature—i.e., true in all Banach spaces (or other families of linear topological spaces)—or results in particular spaces: e.g., Hilbert spaces, L^p spaces, reflexive spaces, etc. The uniform boundedness principle is a typical result of the first type. Results of the second type occur in almost every instance where functional analysis is applied to a specific problem. For example, when dealing with a differential equation, one usually considers a Hilbert space (or L^p, or appropriate Sobolev space) setting and uses the special properties of this space to solve the equation. It is then a different problem (in which, again, the structure of the space usually plays an important role) to prove that the solution found is, in fact, smooth.

With time, linear topological spaces, and Banach spaces in particular, became the subject of an independent study. A beautiful and deep theory emerged, involving the analysis of the structure and classification of general Banach spaces as well as the detailed study of specific Banach spaces, many of which are the common spaces of analysis.

It is not at all surprising that many of the invariants developed for this study had to do with the behavior of sequences and series in the various spaces. Here is a quick early example. A sequence (x_n) in a Banach space is called weak-Cauchy if $\lim x^*(x_n)$ exists for every continuous linear functional x^*. A Banach space is called weakly sequentially complete if every weak-Cauchy sequence in it converges weakly. It is already shown in Banach's *Théorie des opérations linéaires* that $L^1[0,1]$ is weakly sequentially complete, while $C[0,1]$ is not. Thus, one cannot embed $C[0,1]$ as a closed subspace of $L^1[0,1]$.

This is a very simple, yet typical, example. It involves relations between a space and its dual and convergence with respect to one of the natural