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HOMOTOPY GROUPS OF THE COMPLEMENTS TO SINGULAR HYPERSURFACES

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In the early thirties O. Zariski (cf. [**6**, Chapter 8]) discovered a relationship between the fundamental group of the complement to a plane curve in $\mathbf{CP}^2$ and the first Betti number of cyclic covers of $\mathbf{CP}^2$ branched over this curve. At the same time, he determined the precise relationship between this Betti number and the position of singularities of the branching locus. Here we shall describe the relationship between certain higher homotopy groups of the complement to a hypersurface in $\mathbf{CP}^{n+1}$ and Hodge numbers of cyclic covers of $\mathbf{CP}^{n+1}$ branched over this hypersurface, as well as their relation to the position of singularities of the branching locus.

Let V_d^n be a hypersurface of degree d in $\mathbf{CP}^{n+1}$. Let k denote the dimension of the singular locus of V_d^n. We shall fix a generic hyperplane H (i.e., transversal to the strata of the singular locus of V_d^n). Let H_{n-k} denote a generic linear subspace of $\mathbf{CP}^{n+1}$ of dimension $n-k$. It follows from Zariski's theorem ([**2**]) and the computation of homotopy groups of the complement to a nonsingular hypersurface (cf. [**1**]) that

$$(1)\quad \pi_i(\mathbf{CP}^{n+1} - V_d^n) = \pi_i(H_{n-k} - V_d^n) = \begin{cases} \mathbf{Z}/d & i = 1, \\ 0 & i = 2,\ldots,n-k-1, \end{cases}$$

and

$$(2)\qquad \pi_i(\mathbf{CP}^{n+1} - (V_d^n \cup H)) = \begin{cases} \mathbf{Z} & i = 1, \\ 0 & i = 2,\ldots,n-k-1, \end{cases}$$

Therefore the first homotopy group of $\mathbf{CP}^{n+1} - V_d^n$ (resp. $\mathbf{CP}^{n+1} - (V_d^n \cup H)$) which can be affected by singularities of V_d^n is π_{n-k}. From now on we shall consider $\pi_{n-k}(\mathbf{CP}^{n+1} - V_d^n)$ (resp. $\pi_{n-k}(\mathbf{CP}^{n+1} - (V_d^n \cup H))$) as $\mathbf{Z}/d$-module (resp. as $\mathbf{Z}$-module), where the action is the usual action of fundamental group on a homotopy group.

PROPOSITION 1. *Let $d\mathbf{Z}$ denote the subgroup of index d in $\mathbf{Z}$. Then $\pi_{n-k}(\mathbf{CP}^{n+1} - V_d^n)$ is isomorphic to the submodule of invariants*

$$\pi_{n-k}\left(\mathbf{CP}^{n+1} - (V_d^n \cup H)^{\mathrm{Inv}\, d\mathbf{Z}}\right)$$

as $\mathbf{Z}/d$-module.

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