BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 13, Number 1, July 1985

DEHN SURGERY ON KNOTS

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Let M be a compact, connected, orientable, irreducible 3-manifold such that ∂M is a torus. An isotopy class c of unoriented simple closed curves in ∂M will be called a *slope*. A closed 3-manifold $M(c)$ may be constructed by attaching a solid torus J to M so that c bounds a disk in J.

If c and d are two slopes, we denote their (minimal) geometric intersection number by $\Delta(c, d)$.

THEOREM. *Suppose that M is not a Seifert fibered space. If $\pi_1(M(c))$ and $\pi_1(M(d))$ are cyclic, then $\Delta(c, d) \leqslant 1$. In particular, there are at most three slopes c such that $\pi_1(M(c))$ is cyclic.*

This result is sharp; Fintushel–Stern and Berge have given examples of hyperbolic knots in S^3 for which two Dehn surgeries give lens spaces.

In the statements of the following corollaries we use rational numbers as in **[R]** to parametrize the nontrivial Dehn surgeries on a knot K in S^3. We will denote by $K(r)$ the result of r-surgery on K.

COROLLARY 1. *If K is not a torus knot and $r \in \mathbf{Q}$, then $\pi_1(K(r))$ can be cyclic only if r is an integer. Moreover, there are at most two such integers r, and if there are two then they must be successive.*

COROLLARY 2. *If K is a nontrivial knot and $r \in \mathbf{Q}$ is not equal to 1 or -1 then $K(r)$ is not simply-connected. Moreover, $K(1)$ and $K(-1)$ cannot both be simply-connected.*

COROLLARY 3. *Up to unoriented equivalence, there are at most two knots whose complements are of a given topological type.*

COROLLARY 4. *If K is a nontrivial amphicheiral knot and $r \in \mathbf{Q} - \{0\}$, then $\pi_1(K(r))$ is not cyclic. In particular, K has Property P.*

COROLLARY 5. *Knots of Arf invariant 1 are determined up to unoriented equivalence by their complements.*

Whitten **[W]**, using work of Johannson **[Jo1]**, shows that Corollary 1 implies the following result.

COROLLARY 6. *Prime knots with isomorphic groups have homeomorphic complements.*

Received by the editors February 7, 1985.
1980 *Mathematics Subject Classification*. Primary 57M25, 57R65.

0273-0979/85 $1.00 + $.25 per page