an active and creative scientist. Such details are best left to the historians and to the book reviewers, who are usually delighted by the opportunity to fill them in.

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Commutative semigroup rings, by Robert Gilmer, Chicago Lectures in Mathematics, The University of Chicago Press, Chicago, IL, xii + 380 pp., 1984, $11.00. ISBN 0-2262-9392-0

Let R be a commutative ring and S a semigroup with respect to an operation $+$, not necessarily commutative. The semigroup ring $R[X; S]$ consists of formal sums $\sum_{i=1}^{n} r_i X^{s_i}$, $r_i \in R$, $s_i \in S$, with addition defined by adding coefficients, and multiplication defined distributively using the rule $X^s X^t = X^{s+t}$. For example, if N is the semigroup of nonnegative integers, then $R[X; N]$ is just the polynomial ring $R[X]$ in a single indeterminate X. Another important example is the semigroup ring $K[X; G]$, where K is a field and G is a finite group. The theory of semigroup rings divides much along the lines of these two examples. If $R[X]$ is taken as the starting point, then the tools and problems come from commutative algebra; if the starting point is $K[X; G]$, then the group G is the primary object of study, and the tools come from group representation theory and constitute a rich mixture of many other areas of mathematics. It should be emphasized that in the case of $K[X; G]$, the main interest is in a nonabelian group G; indeed, a large portion of noncommutative ring theory has been developed specifically in order to deal with this example. (A nice set of lectures on this aspect of the subject, with the ultimate goal of proving a couple of important theorems on finite groups, can be founnd in [**6**].)

To get an idea of the shift in emphasis imposed by restricting to a commutative semigroup S, as is done in this book, consider the question of semisimplicity of $K[X; G]$. A ring is called semisimple if its Jacobson radical is 0. For a commutative ring A, the Jacobson radical $J(A)$ is defined to be the intersection of the maximal ideals of A. A related notion is that of the nilradical $N(A)$, which is the intersection of the prime ideals of A. The ring A is called a Hilbert ring if every prime ideal is an intersection of maximal ideals, in which case, clearly, $N(A) = J(A)$. The definitions of $J(A)$ and $N(A)$ for a noncommutative ring are somewhat more complicated.

For a finite group G, $K[X; G]$ is semisimple, provided that G has no element of order p when char $K = p > 0$; this is Maschke's theorem and is fundamental for the classification of the representations of G (cf. [**6**, p. 244]). The statement remains true if, instead of being finite, G is taken to be an arbitrary abelian group (cf. [**4**, p. 73, Corollary 17.8]). To what extent does this latter result surface in the present book? The nearest theorem to it that I could find is Theorem 11.14, p. 140, which only yields the case that G is torsion-free. On the