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*Boundary-value problems with free boundaries for elliptic systems of equations*, by V. N. Monakhov, Translations of Mathematical Monographs, Vol. 57, Americal Mathematical Society, Providence, R.I., 1983, xiv + 522 pp, $110.00. ISBN 0-8218-4510-1 (Originally published by "Nauka", Novosibirsk, 1977)

This book breathes new life into a point of view the reviewer had long considered to have expired with his own contribution of 1956. The conceptual starting point comes from the papers of Weinstein (1928–1929), Leray and Weinstein (1934), and Leray (1935–1936). Weinstein developed an earlier suggestion of Weyl to use the continuity method; he proved the unique existence, under some conditions, of a steady two-dimensional incompressible potential flow through a polygonal nozzle with partially free boundaries determined by a constant pressure condition. The result was later improved by Leray and Weinstein, while Leray showed that such problems are, in principle, accessible to fixed-point methods.

All these results imposed (total) curvature conditions on the fixed boundary. The author applies the Weyl-Weinstein procedure to the case of Kirchhoff flow past an obstacle and finds again a unique solution subject to certain curvature conditions. On the other hand, the reviewer (loc. cit.) found analogous results under very different kinds of restrictions; thus it seems likely that the restrictions are intrinsic to the methods and not to the problem. In this sense the continuity method, which is conceptually very appealing, still leaves major questions unsettled.

The author then follows the lead taken by Leray and shows that the Leray-Schauder theorem can be applied to obtain a very general existence result without significant curvature restrictions, but also without a uniqueness or local stability proof. The result is shown to include Kirchhoff flows, Thullen vortex flows, the reentrant jet of Efros, Riabouchinsky and Lavrent'ev flows, flow through a nozzle, and flow in channels with partially unknown walls. Explicit representations for the solutions are given in terms of undetermined parameters.

For many authors this would be a natural stopping point; for this author it became a plateau from which further flights could be initiated. He proceeds at first to show that other kinds of conditions (e.g., gravitational and/or capillary) can be included by an approximation procedure. That brings the reader to about the halfway point in the book. The remainder of the book is devoted to extensions to other flow equations that are governed by quasi-conformal, rather than conformal, mappings. There is a long chapter on mapping theorems for multiply connected domains, by quasi-conformal mappings determined by nonlinear systems; this is followed by a chapter on the Hilbert, and other, boundary value problems and potential and singular operators.