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CLASSIFICATION OF FIRST ORDER THEORIES WHICH HAVE A STRUCTURE THEOREM

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We first explain the problem, then the solution and various consequences; we then discuss the limits and possible criticisms of our solution. Full proofs will appear in [**8**].

Let T denote a countable complete first order theory. A model M of T is a set $|M|$ with interpretations of the predicates and the function symbols appearing in T as relations and functions on $|M|$.

1. The problem. As we view model theory also as an abstract algebra (i.e., dealing with any T, not just a specific one), we want to find a general structure theorem for the class of models of T like those of Steinitz (for algebraically closed fields) and Ulm (for countable torsion abelian groups). So, ideally, for every model M of T we should be able to find a set of invariants which is complete, i.e., determines M up to isomorphism. Such an invariant is the isomorphism type, so we should better restrict ourselves to more reasonable ones, and the natural candidates are cardinal invariants or reasonable generalizations of them. For a vector space over Q we need one cardinal (the dimension); for a vector space over an algebraically closed field, two cardinals; for a divisible abelian group G, countably many cardinals (the dimension of $\{x \in G\colon px = 0\}$ for each prime p and the rank of $G/\mathrm{Tor}(G)$); and for a structure with countably many one-place relations (i.e. $\equiv$ distinguished subsets), we need $2^{\aleph_0}$ cardinals (the cardinality of each Boolean combination).

We believe the reader will agree that every model $(|M|, E)$, where E is an equivalence relation, has a reasonably complete set of invariants: namely, the function saying, for each cardinal λ, how many equivalence classes of this power occur. Also, if we enrich M by additional relations which relate only equivalent members and such that each equivalence class becomes a model with a complete set of invariants, then the resulting model will have a complete set of invariants.

However, even if we allow such generalized cardinal invariants, we cannot have such a structure theory for every T, so we have to reformulate our problem.

1.1. THE STRUCTURE/NONSTRUCTURE PROBLEM. *Describe for some T's a structure theory and prove for the other theories nonstructure theorems showing that no structure theory is possible.*

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