BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 12, Number 2, April 1985

ALGEBRAIC K-THEORY OF POLY-(FINITE OR CYLIC) GROUPS

BY FRANK QUINN

ABSTRACT. The K-theory of the title is described in terms of the K-theory of finite subgroups, as generalized sheaf homology of a quotient space. A corollary is that if G is torsion-free, then the Whitehead groups $\mathrm{Wh}_i(\mathbf{Z}G)$ vanish for all i.

1. The main result. Suppose that G is a poly-(finite or cyclic) group. Then there is a virtually connected and solvable Lie group L that contains G as a discrete cocompact subgroup. ("Virtually" means a subgroup of finite index has the indicated property.) This follows from results of Auslander and Johnson, as observed in [**5**]. Let K be a maximal compact subgroup of L. Then G acts (on the right) on the contractible manifold $K\backslash L$. The action may not be free; for $y \in (K\backslash L)$ the isotropy subgroup is $G_y = (yGy^{-1}) \cap K$. These isotropy subgroups are finite since they are discrete in the compact group K.

Consider the quotient $K\backslash L/G$. Let $[y]$ denote a point with preimage y in $K\backslash L$. Then the isotropy subgroup G_y is determined by $[y]$ up to conjugacy. G_y therefore defines a "cosheaf up to conjugacy" of groups over $K\backslash L/G$. If R is a ring we can apply the algebraic K-theory functor $\mathbf{K}(R[*])$ to this to get a cosheaf of spectra over $K\backslash L/G$. Homology groups with coefficients in a spectral cosheaf are defined [**12**] and, in this case, denoted by $H_i(K\backslash L/G; \mathbf{K}(RG_y))$. Homology is discussed further in §2.

The K-theory spectrum used here is the nonconnective one; the lower homotopy groups of the spectrum are Bass's groups K_{-j}. Also, maps of $\mathbf{K}(RG)$ induced by conjugation of G are essentially canonically homotopic to the identity. Therefore the uncertainty of definition of the G_y vanishes at the spectrum level.

1.1 THEOREM. *Suppose G is a discrete cocompact subgroup of a Lie group L that is virtually connected and solvable and has maximal compact subgroup K. Suppose R is a subring of the rationals in which the order of the torsion of G is invertible. Then the natural homomorphism*

$$H_i(K\backslash L/G; \mathbf{K}(RG_y)) \to K_i(RG)$$

is an isomorphism for all i.

Before discussing the proof we give corollaries and references. First, if G is virtually abelian Yamasaki [**18**] has proved the analog of 1.1 for the surgery groups of G.

Received by the editors October 29, 1984.
1980 *Mathematics Subject Classification*. Primary 16A54, 18F25, 22E40.