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EQUIVARIANT h-COBORDISMS AND FINITENESS OBSTRUCTIONS

BY MARK STEINBERGER AND JAMES WEST[1]

ABSTRACT. We classify up to topological equivalence those equivariant h-cobordisms which admit a handle structure, giving a topologically invariant Whitehead group and an s-cobordism theorem, and giving the comparison to the Diff and PL classification via an equivariant version of the controlled Whitehead groups of Chapman and Quinn. We also construct stably triangulable, compact G-manifolds with boundary which realize arbitrary controlled equivariant finiteness obstructions. The controlled finiteness obstructions of their boundaries are generic for closed G-manifolds whose product with $\mathbf{R}$ is triangulable.

Here G is a finite group, and G-manifolds are assumed to be locally linear with codimension-3 gaps (proper inclusions of fixed-point components have codimension ≥ 3). By a G-h-cobordism on M we mean a proper equivariant h-cobordism which admits a handle structure (equivariant) in which no handles are attached to fixed-point components of M of dimension less than 5.

Let $\mathrm{Wh}_G^{\mathrm{PL}}(M)$ be the locally compact version as in [**Si**] of Illman's Whitehead group [**Il₁**], and let $\mathrm{Wh}_G^{\mathrm{PL},\rho}(M)$, be the subgroup consisting of pairs (Y, M) such that $Y_\alpha^H = M_\alpha^H \cup Y_\alpha^{>H}$ if either $M_\alpha^H = M_\alpha^{>H}$ or $\dim M_\alpha^H < 5$ for each component M_α^H of M^H. With our assumptions, G-h-cobordisms with an explicit handle structure are classified up to handle manipulation (or Cat isomorphism of Cat $=$ Diff or PL) by $\mathrm{Wh}_G^{\mathrm{PL},\rho}(M)$ (cf. [**BQ, R**]). This gives an s-cobordism theorem in Diff and PL, but as noted by [**Il$_2$**, **BH**, **R**] and [**DR**], any G-h-cobordism whose torsion lies in $\mathrm{Wh}_G^{\mathrm{PL}}(x) \subset \mathrm{Wh}_G^{\mathrm{PL}}(M)$, for $x \in M^G$, is topologically trivial.

We say that two G-pairs (Y, X) and (Z, X) are stably homeomorphic if there is a homeomorphism $Y \times Q_G \cong Z \times Q_G$ commuting up to proper G-homotopy with the inclusions of X, where Q_G is the product of infinitely many copies of the unit disc in the regular representation of G over $\mathbf{R}$. The

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