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FINITENESS OF MORDELL–WEIL GROUPS OF GENERIC ABELIAN VARIETIES

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In a series of papers in the 1960s Shimura studied analytic families of abelian varieties with fixed polarization, endomorphism, and level structure. The isomorphism classes of abelian varieties in such a family are in one-to-one correspondence with the points of D/Γ, where D is a symmetric domain and Γ is a discontinuous group of transformations of D. Shimura constructed a fibre system (V, W) where the base V is analytically isomorphic to D/Γ, the fibres are the abelian varieties in the family, and V and W are quasi-projective varieties. The fibre A over the generic point of V is an abelian variety defined over the function field K of V. The main result of this announcement is that, under certain conditions on the endomorphism algebra structure, the group of points of A defined over K is finite. Using completely different techniques, Shioda [**8**] proved this result in the case in which D is the complex upper half-plane and Γ is a congruence subgroup of $\mathrm{SL}_2(\mathbf{Z})$.

The results in this note are an extension of part of the author's Ph.D. thesis [**9**]. Details will appear elsewhere. I would like to express my sincere thanks to my thesis advisor, Professor Goro Shimura.

1. Let F be an arbitrary totally real number field of degree g over the rational number field $\mathbf{Q}$. Let L be either (a) the field F, (b) a totally indefinite quaternion algebra over F (and view L as embedded in $M_2(\mathbf{R})^g$), or (c) a totally imaginary quadratic extension K of F. Let Φ be a representation of L by complex matrices of degree n so that $\Phi + \overline{\Phi}$ is equivalent to a rational representation of L, and $\Phi(1) = 1_n$ (writing 1_n for the identity matrix of size n). Assume that $[L : \mathbf{Q}]$ divides $2n$, and let $m = 2n/[L : \mathbf{Q}]$. In (c), if $\tau_1, \ldots, \tau_g, \overline{\tau}_1, \ldots, \overline{\tau}_g$ are the distinct embeddings of K in the complex number field $\mathbf{C}$, write r_ν and s_ν, respectively, for the multiplicities of τ_ν and $\overline{\tau}_\nu$ in Φ (then $r_\nu + s_\nu = m$). Suppose $T \in M_m(L)$ satisfies ${}^tT^\rho = -T$, where ^{t} is transpose on $M_m(L)$, and ρ is complex conjugation on K and transpose on each factor of $M_2(\mathbf{R})^g$. In (c), suppose iT^{τ_ν} has the same signature as

$$\begin{pmatrix} 1_{r_\nu} & 0 \\ 0 & -1_{s_\nu} \end{pmatrix}$$

for every ν. Let $\mathcal{M}$ be a lattice in L^m, and let $v_1, \ldots, v_s$ be elements of L^m. Let Ω denote the collection of data $(L, \Phi, \rho, T, \mathcal{M}, v_1, \ldots, v_s)$.

Suppose A is an abelian variety with a polarization C, θ is an embedding of L into $\mathrm{End}(A) \otimes \mathbf{Q}$, and $t_1, \ldots, t_s$ are elements of A of finite order.

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