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ON THE CONSTRUCTION OF CODIMENSION-TWO MINIMAL IMMERSIONS OF EXOTIC SPHERES

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1. Introduction. It is, nowadays, a well-known fundamental fact in differential topology that there are many differential manifolds which are *homeomorphic* to but *nondiffeomorphic* to spheres. Such differentiable manifolds were christened *exotic spheres* by their founder, J. Milnor. The structures of those exotic spheres which can be realized as the boundaries of parallelizable manifolds were systematically analyzed by the technique of surgery in [**KM**]. They can be imbedded as codimension-two submanifolds of the standard spheres.

In differential geometry and geometric measure theory, the study of closed minimal submanifolds of the euclidean n-sphere $S^n(1)$ is directly related to that of the local structure of singularities of minimal submanifolds in the general Riemannian setting. Therefore, closed minimal submanifolds of $S^n(1)$ are not only interesting, nice, geometric objects in themselves, but are also of basic, theoretical importance in the study of geometric variational theory. Among various problems on the existence, as well as on the uniqueness, of closed minimal submanifolds of euclidean spheres, the following problems naturally distinguish themselves as especially interesting.

Problem 1. The existence and uniqueness problem of minimal imbeddings (or immersions) of codimension-one spheres in $S^n(1)$ (the spherical Bernstein problem proposed by S. S. Chern [**2**]).

Problem 2. The existence and uniqueness problem of minimal imbeddings (or immersions) of codimension-two exotic or knotted spheres in $S^n(1)$.

We announce here the existence of codimension-two minimal immersions of exotic spheres into euclidean spheres. The following is a brief outline of a method of construction of such examples of minimal immersions of exotic spheres of Kervaire type, Σ_0^{4k+1}, into $S^{4k+3}(1)$.

2. The Basic ideas of construction. In the study of the *degree of symmetry* of exotic spheres [**4**], one finds that the Kervaire sphere $\Sigma_0^{4k+1} \in bP_{4k+2}$ is the most symmetric exotic sphere, namely,

$$\begin{aligned} N(\Sigma^m) &= \max\{\dim G\colon G \text{ compact subgroup of } \mathrm{Diff}(\Sigma^m)\} \\ &\leq \tfrac{1}{8}(m^2+7) \end{aligned}$$

for all exotic m-spheres Σ^m, and, in the above, equality holds when and only when $\Sigma^m = \Sigma_0^{4k+1} \in bP_{4k+2}$. Compact, differentiable transformation groups on Σ_0^{4k+1} with the above highest possible dimension, i.e., $\dim G = \frac{1}{8}(m^2+7) = k(2k+1)+1$, are classified in [**3**]. It is a rather pleasant surprise

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