BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 12, Number 1, January 1985

RIGIDITY OF SOME TRANSLATIONS ON HOMOGENEOUS SPACES

BY DAVE WITTE

Ornstein and Weiss [**2**] proved that the geodesic flow on a compact surface of constant negative curvature manifests extreme randomness (it is Bernoulli). In contrast, Ratner [**3**] has shown that the horocycle flow is very rigid: a measure-theoretic isomorphism between two horocycle flows must be an affine map (a.e.). More concretely, suppose Γ and Λ are lattices in $G = \mathrm{SL}_2(\mathbf{R})$, and let $u = \left(\begin{smallmatrix} 1 & t \\ 0 & 1 \end{smallmatrix}\right)$ be a (nonidentity) unipotent element of G. Ratner showed that if $\psi\colon G/\Gamma \to G/\Lambda$ is a u-equivariant measure-preserving Borel map, then ψ is an affine map (a.e.). Here we announce the proof of a satisfactory extension of this rigidity theorem to the situation where G is replaced by any connected semisimple Lie group.

A discrete subgroup Γ of G is a *lattice* if the homogeneous space G/Γ has finite volume. We say $\psi\colon G/\Gamma \to H/\Lambda$ is *affine for* $g \in G$ if there is some $h \in H$ with $\psi(\Gamma x g) = \psi(\Gamma x)\cdot h$ for a.e. $\Gamma x \in G/\Gamma$. Obviously, ψ is affine for g if ψ is *affine for* G: i.e., if ψ is affine for each element of G. The subject of this announcement is a converse to this statement—for translations of zero entropy (this includes the unipotent translations). Note that if H acts faithfully on H/Λ, then any measure-preserving map $\psi\colon G/\Gamma \to H/\Lambda$ that is affine for G is an affine map (a.e.); i.e., there is a continuous surjective homomorphism $\sigma\colon G \to H$ and some $h_0 \in H$ such that $\psi(\Gamma x) = \Lambda h_0 \cdot \sigma(x)$ for a.e. $\Gamma x \in G/\Gamma$.

THEOREM. *Suppose Γ and Λ are lattices in connected semisimple Lie groups G and H. If $\psi\colon G/\Gamma \to H/\Lambda$ is a measure-preserving Borel map that is affine for a zero entropy ergodic translation of G/Γ, then ψ is affine for G.*

It is easy to prove the theorem under a hypothesis of high $\mathbf{R}$-rank. We begin by showing that if ψ is affine for a zero entropy ergodic translation g, then ψ is affine for every element of the connected centralizer $C_G(g)^0$. (This is based on polynomial divergence of orbits—the argument is due to Ratner.) Repeating the argument, we know ψ is affine for centralizers of ergodic unipotent elements of $C_G(g)^0$, and for centralizers of unipotent elements of these centralizers, and so on. In high $\mathbf{R}$-rank we eventually reach a collection of centralizers that generate G—thus ψ is affine for G. The case of low $\mathbf{R}$-rank takes more work. Because the centralizers do not generate, we need to base arguments on commutation relations satisfied by elements of subgroups of G (cf. the proof of Lemma 3.4 in [**3**]).

When G and H are connected noncompact simple Lie groups with finite center, we can construct ergodic G-actions by embedding G in H: then G acts

Received by the editors April 23, 1984.
1980 *Mathematics Subject Classification*. Primary 58F11; Secondary 22D40, 28D20.

0273-0979/85 $1.00 + $.25 per page