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SOME RECENT DEVELOPMENTS IN FOURIER ANALYSIS AND H^p-THEORY ON PRODUCT DOMAINS[1]

BY SUN-YUNG A. CHANG AND ROBERT FEFFERMAN

Introduction. In this article we wish to discuss a theory which is still developing very rapidly. It is only quite recently that many of the aspects of Fourier analysis of several parameters have been discovered, even though much of the corresponding one-parameter theory has been well known for some time. The topics to be covered include differentiation theory, singular integrals, Littlewood–Paley theory, weighted norm inequalities, Hardy spaces, and functions of bounded mean oscillation, as well as many other related topics. We shall begin in Part I by attempting to give a broad overview of some of the one-parameter results about these topics. The discussion here is, however, anything but encyclopedic. (For more detailed treatments of these matters in the one-parameter setting, the reader can consult such excellent treatments as E. M. Stein, *Singular integrals and differentiability properties of functions* [**75**], R. R. Coifman and G. Weiss, *Extensions of Hardy spaces and their use in analysis* [**30**], and, in the classical domain of the disc, D. Sarason, *Function theory on the unit circle* [**72**], and J. Garnett, *Bounded analytic functions* [**46**].) In Part II we take up these same areas in the two-parameter setting. Since this theory is less well known than the material of Part I, we go into greater detail and devote separate sections to each of several of the above topics.

PART I. THE ONE-PARAMETER THEORY

To begin with the one-parameter theory, perhaps the most basic part is the differentiation of integrals and the maximal function of Hardy–Littlewood. If f is a function on R^n which is Lebesgue integrable, and if

$$A_r(f)(x) = \frac{1}{m(B_r(x))} \int_{B_r(x)} f(y)\, dy$$

denotes the average value of f over the ball with center x and radius r, then

$$\lim_{r \to 0} A_r(f)(x) = f(x) \quad \text{for a.e. } x \in R^n.$$

This fundamental result of Lebesgue, proved in the earlier years of the century, was applied immediately in a number of contexts. For example, Lebesgue saw that it could be used to show that for integrable functions of one variable, the arithmetic means of the partial sums of the Fourier series converge pointwise almost everywhere.

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